Measurement Systems Analysis

MSA

Fourth Edition

MEASUREMENT SYSTEMS ANALYSIS

Reference Manual

Fourth Edition

First Edition, October 1990 • Second Edition, February 1995; Second Printing, June 1998
Third Edition, March 2002; Second Printing, May 2003; Fourth Edition, June 2010

ISBN#: 978-1-60-534211-5

FOREWORD

This Reference Manual was developed by a Measurement Systems Analysis (MSA) Work Group, sanctioned by the Chrysler Group LLC, Ford Motor Company, and General Motors Corporation Supplier Quality Requirements Task Force, and under the auspices of the Automotive Industry Action Group (AIAG). The Work Group responsible for this Fourth Edition were Michael Down (General Motors Corporation), Frederick Czubak (Chrysler Group LLC), Gregory Gruska (Omnex), Steve Stahley (Cummins, Inc.) and David Benham.

The manual is an introduction to measurement system analysis. ***It is not intended to limit evolution of analysis methods suited to particular processes or commodities.*** While these guidelines are intended to cover normally occurring measurement system situations, there will be questions that arise. These questions should be directed to your authorized customer representative.

June 2010

MSA 4th Edition Quick Guide

Type of Measurement System	MSA Methods	Chapter
Basic Variable	Range, Average & Range, ANOVA, Bias, Linearity, Control Charts	III
Basic Attribute	Signal Detection, Hypothesis Test Analyses	III
Non-Replicable (e.g., Destructive Tests)	Alternate Approaches	IV
Complex Variable	Range, Average & Range, ANOVA, Bias Linearity Control Charts	III, IV
Multiple Systems, Gages or Test Stands	Control Charts ANOVA Regression Analysis	III, IV
Miscellaneous	Alternate Approaches	IV
Other	White Papers – available at AIAG website (www.aiag.org)	

NOTE: Regarding the use of the *GRR* standard deviation

Historically, by convention, a 99% spread has been used to represent the "full" spread of measurement error, represented by a 5.15 multiplying factor (where σ_{GRR} is multiplied by 5.15 to represent a total spread of 99%).

A 99.73% spread is represented by a multiplier of 6.0, which is $\pm 3\sigma$ and represents the full spread of a "normal" curve.

If the reader chooses to increase the coverage level, or spread, of the total measurement variation to 99.73%, use 6.0 as a multiplier in place of 5.15 in the calculations.

Note: *The approach used in the 4th Edition is to compare standard deviations. This is equivalent to using the multiplier of 6 in the historical approach.*

Awareness of which multiplying factor is used is crucial to the integrity of the equations and resultant calculations. This is especially important if a comparison is to be made between measurement system variability and the tolerance. Consequently, if an approach other than that described in this manual is used, a statement of such must be stated clearly in any results or summaries (particularly those provided to the customer).

TABLE OF CONTENTS

List of Tables

List of Figures

CHAPTER I

General Measurement System Guidelines

Section A
Introduction, Purpose and Terminology

Introduction

Measurement data are used more often and in more ways than ever before. For instance, the decision to adjust a manufacturing process is now commonly based on measurement data. The data, or some statistic calculated from them, are compared with statistical control limits for the process, and if the comparison indicates that the process is out of statistical control, then an adjustment of some kind is made. Otherwise, the process is allowed to run without adjustment. Another use of measurement data is to determine if a significant relationship exists between two or more variables. For example, it may be suspected that a critical dimension on a molded plastic part is related to the temperature of the feed material. That possible relationship could be studied by using a statistical procedure called regression analysis to compare measurements of the critical dimension with measurements of the temperature of the feed material.

Studies that explore such relationships are examples of what Dr. W. E. Deming called *analytic studies*. In general, an analytic study is one that increases knowledge about the system of causes that affect the process. Analytic studies are among the most important uses of measurement data because they lead ultimately to better understanding of processes.

The benefit of using a data-based procedure is largely determined by the quality of the measurement data used. If the data quality is low, the benefit of the procedure is likely to be low. Similarly, if the quality of the data is high, the benefit is likely to be high also.

To ensure that the benefit derived from using measurement data is great enough to warrant the cost of obtaining it, attention needs to be focused on the quality of the data.

Quality of Measurement Data

The quality of measurement data is defined by the statistical properties of multiple measurements obtained from a measurement system operating under stable conditions. For instance, suppose that a measurement system, operating under stable conditions, is used to obtain several measurements of a certain characteristic. If the measurements are all “close” to the master value for the characteristic, then the quality of the data is said to be “high”. Similarly, if some, or all, of the measurements are “far away” from the master value, then the quality of the data is said to be “low”.

The statistical properties most commonly used to characterize the quality of data are the *bias* and *variance* of the measurement system. The property called bias refers to the location of the data relative to a reference (master) value, and the property called variance refers to the spread of the data.

One of the most common reasons for low-quality data is too much variation. Much of the variation in a set of measurements may be due to the *interaction* between the measurement system and its environment. For instance, a

measurement system used to measure the volume of liquid in a tank may be sensitive to the ambient temperature of the environment in which it is used. In that case, variation in the data may be due either to changes in the volume or to changes in the ambient temperature. That makes interpreting the data more difficult and the measurement system, therefore, less desirable.

If the interaction generates too much variation, then the quality of the data may be so low that the data are not useful. For example, a measurement system with a large amount of variation may not be appropriate for use in analyzing a manufacturing process because the measurement system's variation may mask the variation in the manufacturing process. Much of the work of managing a measurement system is directed at monitoring and controlling variation. Among other things, this means that emphasis needs to be placed on learning how the measurement system interacts with its environment so that only data of acceptable quality are generated.

Purpose

The purpose of this document is to present guidelines for assessing the quality of a measurement system. Although the guidelines are general enough to be used for any measurement system, they are intended primarily for the measurement systems used in the industrial world. This document is not intended to be a compendium of analyses for all measurement systems. Its primary focus is measurement systems where the readings can be replicated on each part. Many of the analyses are useful with other types of measurement systems and the manual does contain references and suggestions. It is recommended that competent statistical resources be consulted for more complex or unusual situations not discussed here. Customer approval is required for measurement systems analysis methods not covered in this manual.

Terminology

The discussion of the analysis of measurement system can become confusing and misleading without an established set of terms to refer to the common statistical properties and related elements of the measurement system. This section provides a summary of such terms which are used in this manual.

In this document, the following terms are used:

- **Measurement** is defined as "the assignment of numbers [or values] to material things to represent the relations among them with respect to particular properties." This definition was first given by C. Eisenhart (1963). The process of assigning the numbers is defined as the measurement process, and the value assigned is defined as the measurement value.

- **Gage** is any device used to obtain measurements; frequently used to refer specifically to the devices used on the shop floor; includes go/no-go devices (also, see Reference List: ASTM E456-96).
- **Measurement System** is the collection of instruments or gages, standards, operations, methods, fixtures, software, personnel, environment and assumptions used to quantify a unit of measure or fix assessment to the feature characteristic being measured; the complete process used to obtain measurements.

From these definitions it follows that a measurement process may be viewed as a manufacturing process that produces numbers (data) for its output. Viewing a measurement system this way is useful because it allows us to bring to bear all the concepts, philosophy, and tools that have already demonstrated their usefulness in the area of statistical process control.

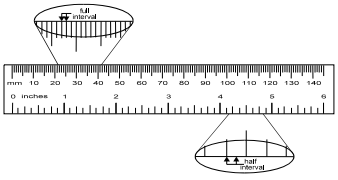
Summary of Terms[1]

Standard

- Accepted basis for comparison
- Criteria for acceptance
- Known value, within stated limits of uncertainty, accepted as a true value
- Reference value

> A standard should be an operational definition: a definition which will yield the same results when applied by the supplier or customer, with the same meaning yesterday, today, and tomorrow.

Basic equipment



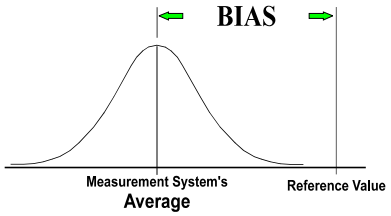
- **Discrimination, readability, resolution**
  - ✓ Alias: smallest readable unit, measurement resolution, scale limit, or detection limit
  - ✓ An inherent property fixed by design
  - ✓ Smallest scale unit of measure or output for an instrument
  - ✓ Always reported as a unit of measure
  - ✓ 10 to 1 rule of thumb
- **Effective resolution**
  - ✓ The sensitivity of a measurement system to process variation for a particular application

[1] See Chapter I, Section E for terminology definitions and discussion.

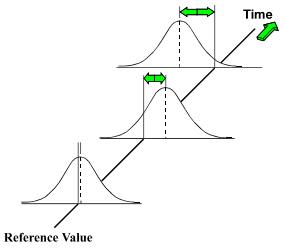
- ✓ Smallest input that results in a usable output signal of measurement
- ✓ Always reported as a unit of measure

- **Reference value**
  - ✓ Accepted value of an artifact
  - ✓ Requires an operational definition
  - ✓ Used as the surrogate for the true value
- **True value**
  - ✓ Actual value of an artifact
  - ✓ Unknown and unknowable

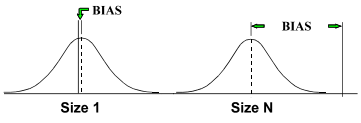
Location variation



- **Accuracy**
  - ✓ "Closeness" to the true value, or to an accepted reference value
  - ✓ ASTM includes the effect of location and width errors
- **Bias**
  - ✓ Difference between the observed average of measurements and the reference value
  - ✓ A systematic error component of the measurement system



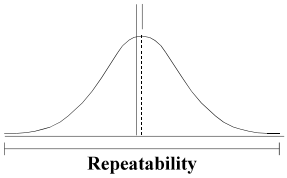
- **Stability**
  - ✓ The change in bias over time
  - ✓ A stable measurement process is in statistical control with respect to location
  - ✓ Alias: Drift



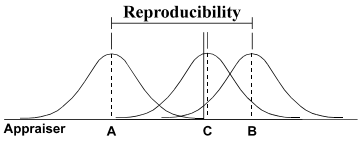
- **Linearity**
  - ✓ The change in bias over the normal operating range
  - ✓ The correlation of multiple and independent bias errors over the operating range
  - ✓ A systematic error component of the measurement system

Width variation

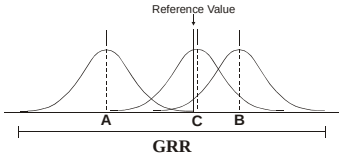
- **Precision**[2]
  - ✓ "Closeness" of repeated readings to each other
  - ✓ A random error component of the measurement system



- **Repeatability**
  - ✓ Variation in measurements obtained with one measuring instrument when used several times by an appraiser while measuring the identical characteristic on the same part
  - ✓ The variation in successive (short-term) trials under fixed and defined conditions of measurement
  - ✓ Commonly referred to as E.V. – Equipment Variation
  - ✓ Instrument (gage) capability or potential
  - ✓ Within-system variation



- **Reproducibility**
  - ✓ Variation in the average of the measurements made by different appraisers using the same gage when measuring a characteristic on one part
  - ✓ For product and process qualification, error may be appraiser, environment (time), or method
  - ✓ Commonly referred to as A.V. – Appraiser Variation
  - ✓ Between-system (conditions) variation
  - ✓ ASTM E456-96 includes repeatability, laboratory, and environmental effects as well as appraiser effects



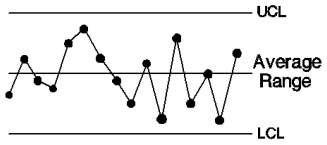
- ***GRR* or Gage R&R**
  - ✓ Gage repeatability and reproducibility: the combined estimate of measurement system repeatability and reproducibility
  - ✓ Measurement system capability; depending on the method used, may or may not include the effects of time

- **Measurement System Capability**
  - ✓ Short-term estimate of measurement system variation (e.g., "*GRR*" including graphics)

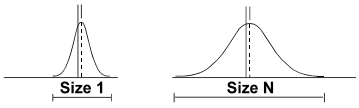
[2] In ASTM documents, there is no such thing as ***the*** precision of a measurement system; i.e., ***the*** precision cannot be represented by a single number.

- **Measurement System Performance**
  - ✓ Long-term estimate of measurement system variation (e.g., long-term Control Chart Method)

- **Sensitivity**
  - ✓ Smallest input that results in a detectable output signal
  - ✓ Responsiveness of the measurement system to changes in measured feature
  - ✓ Determined by gage design (discrimination), inherent quality (Original Equipment Manufacturer), in-service maintenance, and operating condition of the instrument and standard
  - ✓ Always reported as a unit of measure



- **Consistency**
  - ✓ The degree of change of repeatability over time
  - ✓ A consistent measurement process is in statistical control with respect to width (variability)



- **Uniformity**
  - ✓ The change in repeatability over the normal operating range
  - ✓ Homogeneity of repeatability

System variation

Measurement system variation can be characterized as:

- **Capability**
  - ✓ Variability in readings taken over a short period of time

- **Performance**
  - ✓ Variability in readings taken over a long period of time
  - ✓ Based on total variation

- **Uncertainty**
  - ✓ An estimated range of values about the measured value in which the true value is believed to be contained

The measurement system must be stable and consistent.

All characterizations of the total variation of the measurement system assume that the system is stable and consistent. For example, the components of variation can include any combination of the items shown in I-B 1.

Standards and Traceability

The National Institute of Standards and Technology (NIST) is the principal National Measurements Institute (NMI) in the United States serving under the U.S. Department of Commerce. NIST, formerly the National Bureau of Standards (NBS), serves as the highest level authority for metrology in the U.S. NIST's primary responsibility is to provide measurement services and maintain measurement standards that assist U.S. industry in making traceable measurements which ultimately assist in trade of products and services. NIST provides these services directly to many types of industries, but primarily to those industries that require the highest level of accuracy for their products and that incorporate state-of-the-art measurements in their processes.

National Measurement Institutes

Most of the industrialized countries throughout the world maintain their own NMIs and similar to NIST, they also provide a high level of metrology standards or measurement services for their respective countries. NIST works collaboratively with these other NMIs to assure measurements made in one country do not differ from those made in another. This is accomplished through Mutual Recognition Arrangements (MRAs) and by performing *interlaboratory comparisons* between the NMIs. One thing to note is that the capabilities of these NMIs will vary from country to country and not all types of measurements are compared on a regular basis, so differences can exist. This is why it is important to understand to whom measurements are traceable and how traceable they are.

Traceability

Traceability is an important concept in the trade of goods and services. Measurements that are traceable to the same or similar standards will agree more closely than those that are not traceable. This helps reduce the need for re-test, rejection of good product, and acceptance of bad product.

Traceability is defined by the *ISO International Vocabulary of Basic and General Terms in Metrology* (VIM) as:

"The property of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties."

The traceability of a measurement will typically be established through a chain of comparisons back to the NMI. However, in many instances in industry, the traceability of a measurement may be linked back to an agreed upon reference value or "consensus standard" between a customer and a supplier. The traceability linkage of these consensus standards to the NMI may not always be clearly understood, so ultimately it is critical that the measurements are traceable to the extent that satisfies customer needs. With the advancement in measurement technologies and the usage of state-of-the-art measurement systems in industry, the definition as to where and how a measurement is traceable is an ever-evolving concept.

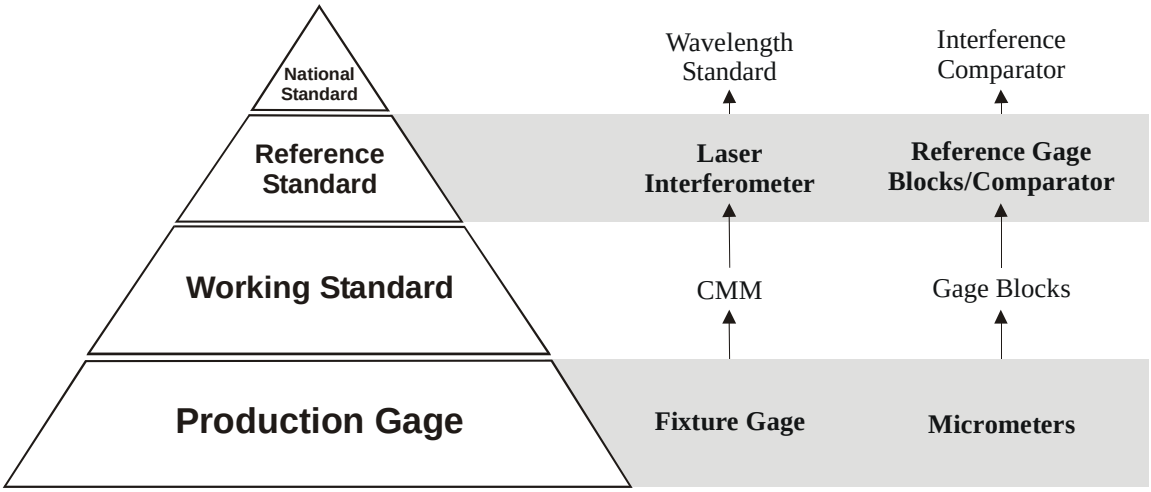


Figure I-A 1: Example of a Traceability Chain for a Length Measurement

NMIs work closely with various national labs, gage suppliers, state-of-the-art manufacturing companies, etc. to assure that their reference standards are properly calibrated and directly traceable to the standards maintained by the NMI. These government and private industry organizations will then use their standards to provide calibration and measurement services to their customers' metrology or gage laboratories, calibrating *working* or other *primary* standards. This linkage or chain of events ultimately finds its way onto the factory floor and then provides the basis for measurement traceability. Measurements that can be connected back to NIST through this unbroken chain of measurements are said to be *traceable to NIST*.

Not all organizations have metrology or gage laboratories within their facilities therefore depend on outside commercial/independent laboratories to provide traceability calibration and measurement services. This is an acceptable and appropriate means of attaining traceability to NIST, provided that the capability of the commercial/independent laboratory can be assured through processes such as laboratory accreditation.

Calibration Systems

A calibration system is a set of operations that establish, under specified conditions, the relationship between a measuring device and a traceable standard of known reference value and uncertainty. Calibration may also include steps to detect, correlate, report, or eliminate by adjustment any discrepancy in accuracy of the measuring device being compared.

The calibration system determines measurement traceability to the measurement systems through the use of calibration methods and standards.

Traceability is the chain of calibration events originating with the calibration standards of appropriate metrological capability or measurement uncertainty. Each calibration event includes all of the elements necessary including standards, measurement and test equipment being verified, calibration methods and procedures, records, and qualified personnel.

An organization may have an internal calibration laboratory or organization which controls and maintains the elements of the calibration events. These internal laboratories will maintain a laboratory scope which lists the specific calibrations they are capable of performing as well as the equipment and methods/procedures used to perform the calibrations.

The calibration system is part of an organization's quality management system and therefore should be included in any internal audit requirements.

Measurement Assurance Programs (MAPs) can be used to verify the acceptability of the measurement processes used throughout the calibration system. Generally MAPs will include verification of a measurement system's results through a secondary independent measurement of the same feature or parameter. Independent measurements imply that the traceability of the secondary measurement process is derived from a separate chain of calibration events from those used for the initial measurement. MAPs may also include the use of statistical process control (SPC) to track the long-term stability of a measurement process.

Note: ANSI/NCSL Z540.3 and ISO 10012 each provide models for many of the elements of a calibration system.

When the calibration event is performed by an external, commercial, or independent calibration service supplier, the service supplier's calibration system can (or may) be verified through accreditation to ISO/IEC 17025. When a qualified laboratory is not available for a given piece of equipment, calibration services may be performed by the equipment manufacturer.

True Value

The measurement process TARGET is the "true" value of the part. It is desired that any individual reading be as close to this value as (economically) possible. Unfortunately, the *true value* can never be known with certainty. However, uncertainty can be minimized by using a *reference value* based on a well defined operational definition of the characteristic, and using the results of a measurement system that has higher order discrimination and traceable to NIST. Because the reference value is used as a surrogate for the true value, these terms are commonly used interchangeably. This usage is not recommended.

Section B
The Measurement Process[3]

Measurement Systems

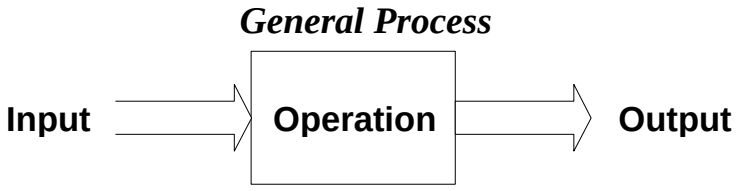
In order to effectively manage variation of any process, there needs to be knowledge of:

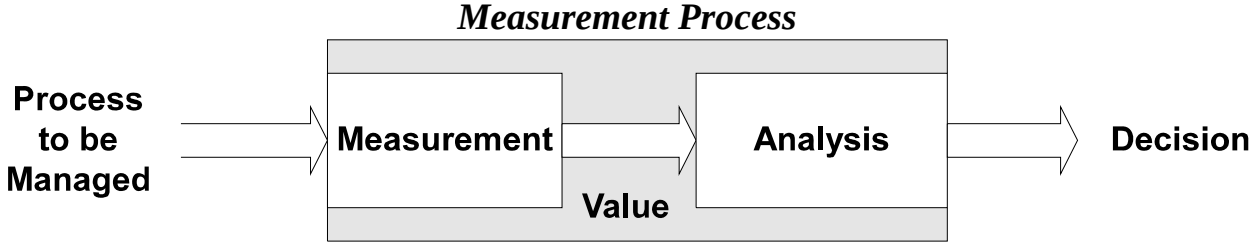
- What the process should be doing
- What can go wrong
- What the process is doing

Specifications and engineering requirements define what the process should be doing.

The purpose of a Process Failure Mode Effects Analysis[4] (PFMEA) is to define the risk associated with potential process failures and to propose corrective action before these failures can occur. The outcome of the PFMEA is transferred to the control plan.

Knowledge is gained of what the process is doing by evaluating the parameters or results of the process. This activity, often called inspection, is the act of examining process parameters, in-process parts, assembled subsystems, or complete end products with the aid of suitable standards and measuring devices which enable the observer to confirm or deny the premise that the process is operating in a stable manner with acceptable variation to a customer designated target. But this examination activity is itself a process.





Unfortunately, industry has traditionally viewed the measurement and analysis activity as a “black box”. Equipment was the major focus – the more "important" the characteristic, the more expensive the gage. The

[3] Portions of this chapter adapted with permission from *Measurement Systems Analysis - A Tutorial* by G. F. Gruska and M. S. Heaphy, The Third Generation, 1987, 1998.

[4] See the *Potential Failure Mode and Effects Analysis (FMEA) Reference Manual – 4th Edition*.

usefulness of the instrument, its compatibility with the process and environment, and its usability was rarely questioned. Consequently these gages were often not used properly or simply not used.

The measurement and analysis activity is a process – a *measurement* process. Any and all of the management, statistical, and logical techniques of process control can be applied to it.

This means that the customers and their needs must first be identified. The customer, the owner of the process, wants to make a correct decision with minimum effort. Management must provide the resources to purchase equipment which is necessary and sufficient to do this. But purchasing the best or the latest measurement technology will not necessarily guarantee correct production process control decisions.

Equipment is only one part of the measurement process. The owner of the process must know how to correctly use this equipment and how to analyze and interpret the results. Management must therefore also provide clear operational definitions and standards as well as training and support. The owner of the process has, in turn, the obligation to monitor and control the measurement process to assure stable and correct results which includes a total measurement systems analysis perspective – the study of the gage, procedure, user, and environment; i.e., normal operating conditions.

Statistical Properties of Measurement Systems

An ideal measurement system would produce only "correct" measurements each time it is used. Each measurement would always agree with a standard.[5] A measurement system that could produce measurements like that would be said to have the statistical properties of zero variance, zero bias, and zero probability of misclassifying any product it measured. Unfortunately, measurement systems with such desirable statistical properties seldom exist, and so process managers are typically forced to use measurement systems that have less desirable statistical properties. The quality of a measurement system is usually determined solely by the statistical properties of the data it produces over time. Other properties, such as cost, ease of use, etc., are also important in that they contribute to the overall desirability of a measurement system. But it is the *statistical properties* of the data produced that determine the quality of the measurement system.

Statistical properties that are most important for one use are not necessarily the most important properties for another use. For instance, for some uses of a coordinate measuring machine (CMM), the most important statistical properties are "small" bias and variance. A CMM with those properties will generate measurements that are "close" to the certified values of standards that are traceable. Data obtained from such a machine can be very useful for analyzing a manufacturing process. But, no matter how "small" the bias and variance of the CMM may be, the measurement system which uses the CMM may be unable to do an acceptable job of discriminating between good and bad product because of the additional sources of variation introduced by the other elements of the measurement system.

[5] For a fuller discussion on the matter of standards see *Out of the Crisis*, W. Edwards Deming, 1982, 1986, p. 279-281.

Management has the responsibility for identifying the statistical properties that are the most important for the ultimate use of the data. Management is also responsible for ensuring that those properties are used as the basis for selecting a measurement system. To accomplish this, operational definitions of the statistical properties, as well as acceptable methods of measuring them, are required. Although each measurement system may be required to have different statistical properties, **there are certain fundamental properties that define a "good" measurement system. These include:**

1) Adequate discrimination and sensitivity. The increments of measure should be small relative to the process variation or specification limits for the purpose of measurement. The commonly known Rule of Tens, or 10-to-1 Rule, states that instrument discrimination should divide the tolerance (or process variation) into ten parts or more. This rule of thumb was intended as a practical minimum starting point for gage selection.
2) The measurement system ought to be in statistical control.FP6PF This means that under repeatable conditions, the variation in the measurement system is due to common causes only and not due to special causes. This can be referred to as statistical stability and is best evaluated by graphical methods.
3) For product control, variability of the measurement system must be small compared to the specification limits. Assess the measurement system to the feature tolerance.
4) For process control, the variability of the measurement system ought to demonstrate effective resolution and be small compared to manufacturing process variation. Assess the measurement system to the 6-sigma process variation and/or Total Variation from the MSA study.

> The statistical properties of the measurement system may change as the items being measured vary. If so, then the largest (worst) variation of the measurement system is small relative to the smaller of either the process variation or the specification limits.

Sources of Variation

Similar to all processes, the measurement system is impacted by both random and systematic sources of variation. These sources of variation are due to common and special causes. In order to control the measurement system variation:

1) Identify the potential sources of variation.
2) Eliminate (whenever possible) or monitor these sources of variation.

Although the specific causes will depend on the situation, some typical sources of variation can be identified. There are various methods of

[6] The measurement analyst must always consider practical and statistical significance.

presenting and categorizing these sources of variation such as cause-effect diagrams, fault tree diagrams, etc., but the guidelines presented here will focus on the major elements of a measuring system.

S	Standard
W	Workpiece (i.e., part)
I	Instrument
P	Person / Procedure
E	Environment

The acronym S.W.I.P.E.[7] is used to represent the six essential elements of a generalized measuring system to assure attainment of required objectives. S.W.I.P.E. stands for Standard, Workpiece, Instrument, Person and Procedure, and Environment. This may be thought of as an error model for a complete measurement system.[8]

Factors affecting those six areas need to be understood so they can be controlled or eliminated.

Figure I-B 1 displays a cause and effect diagram showing some of the potential sources of variation. Since the actual sources of variation affecting a specific measurement system will be unique to that system, **this figure is presented as a thought starter for developing a measurement system's sources of variation**.

[7] This acronym was originally developed by Ms. Mary Hoskins, a metrologist associated with Honeywell, Eli Whitney Metrology Lab and the Bendix Corporation.
[8] See Appendix F for an alternate error model, P.I.S.M.O.E.A.

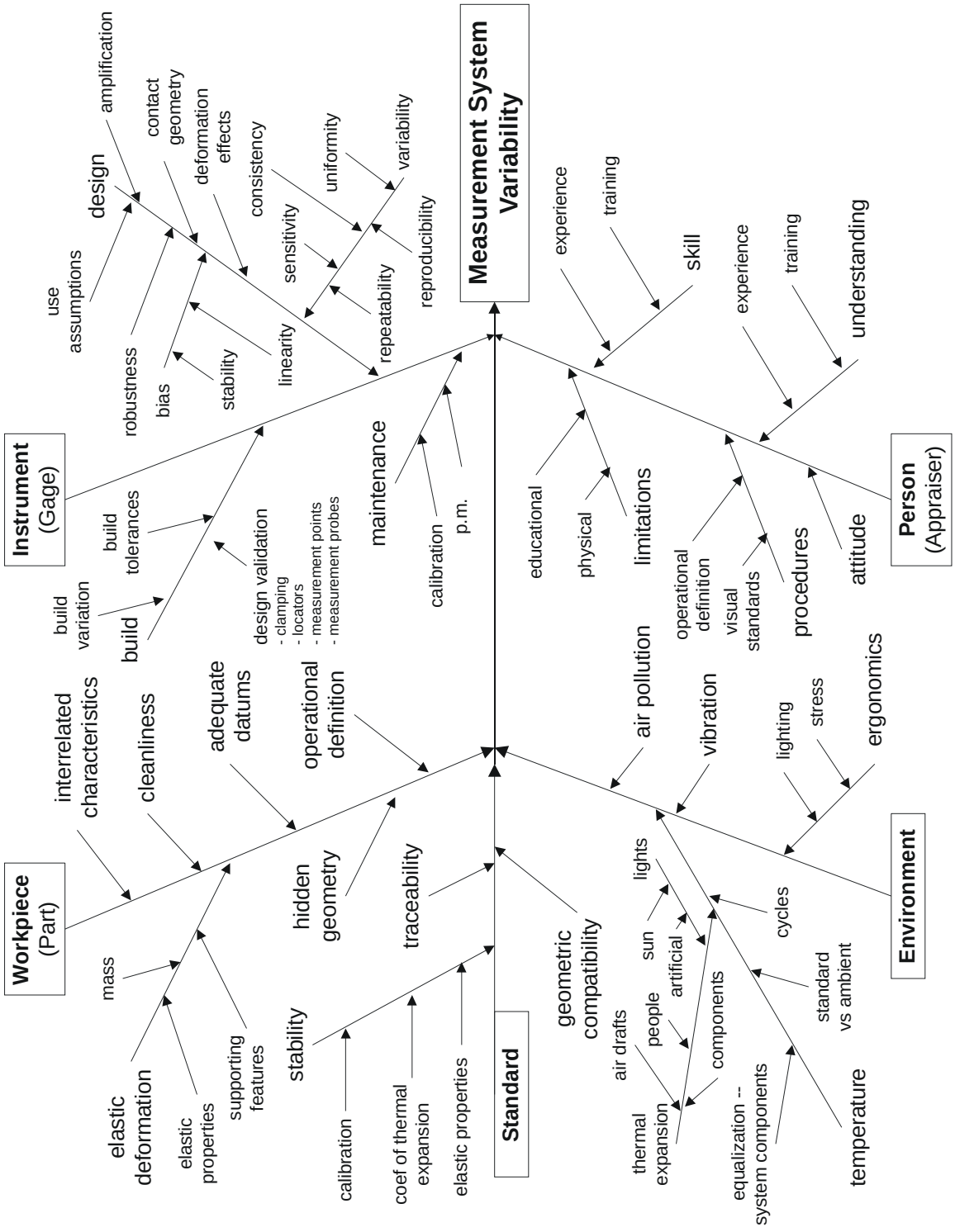


Figure I-B 1: Measurement System Variability Cause and Effect Diagram

The Effects of Measurement System Variability

Because the measurement system can be affected by various sources of variation, repeated readings on the same part do not yield the same, identical result. Readings vary from each other due to common and special causes.

The effects of the various sources of variation on the measurement system should be evaluated over a short and long period of time. The *measurement system capability* is the measurement system (random) error over a short period of time. It is the combination of errors quantified by linearity, uniformity, repeatability and reproducibility. The *measurement system performance*, as with process performance, is the effect of all sources of variation over time. This is accomplished by determining whether our process is in statistical control (i.e., stable and consistent; variation is due only to common causes), on target (no bias), and has acceptable variation (gage repeatability and reproducibility (*GRR*)) over the range of expected results. This adds stability and consistency to the measurement system capability.

> Because the output of the measurement system is used in making a decision about the product and the process, the cumulative effect of all the sources of variation is often called *measurement system error*, or sometimes just "error."

Effect on Decisions

After measuring a part, one of the actions that can be taken is to determine the status of that part. Historically, it would be determined if the part were acceptable (within specification) or unacceptable (outside specification). Another common scenario is the classification of parts into specific categories (e.g., piston sizes).

> For the rest of the discussion, as an example, the two category situation will be used: out of specification ("bad") and in specification ("good"). This does not restrict the application of the discussion to other categorization activities.

Further classifications may be reworkable, salvageable or scrap. Under a *product control philosophy* this classification activity would be the primary reason for measuring a part. But, with a *process control philosophy*, interest is focused on whether the part variation is due to common causes or special causes in the process.

Philosophy	Interest
Product control	Is the part in a specific category?
Process control	Is the process variation stable and acceptable?

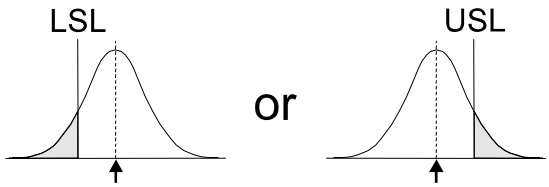
Table I-B1: Control Philosophy and Driving Interest

The next section deals with the effect of the measurement error on the product decision. Following that is a section which addresses its impact on the process decision.

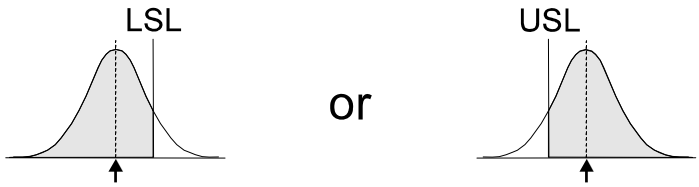
Effect on Product Decisions

In order to better understand the effect of measurement system error on product decisions, consider the case where all of the variability in multiple readings of a single part is due to the gage repeatability and reproducibility. That is, the measurement process is in statistical control and has zero bias.

A wrong decision will sometimes be made whenever any part of the above measurement distribution overlaps a specification limit. For example, a good part will sometimes be called "bad" (type I error, producer's risk or *false alarm*) if:



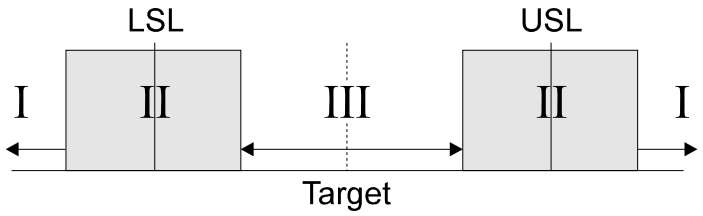
And, a bad part will sometimes be called "good" (type II error, consumer's risk or *miss rate*) if:



NOTE: False Alarm Rate + Miss Rate = *Error Rate*.

RISK is the chance of making a decision which will be detrimental to an individual or process

That is, with respect to the specification limits, the potential to make the wrong decision about the part exists only when the measurement system error intersects the specification limits. This gives three distinct areas:



where:

I	Bad parts will always be called bad
II	Potential wrong decision can be made
III	Good parts will always be called good

Since the goal is to maximize CORRECT decisions regarding product status, there are two choices:

1) Improve the production process: reduce the variability of the process so that no parts will be produced in the II or "shaded" areas of the graphic above.
2) Improve the measurement system: reduce the measurement system error to reduce the size of the II areas so that all parts being produced will fall within area III and thus minimize the risk of making a wrong decision.

This discussion assumes that the measurement process is in statistical control and on target. If either of these assumptions is violated then there is little confidence that any observed value would lead to a correct decision.

Effect on Process Decisions

With process control, the following needs to be established:

- Statistical control
- On target
- Acceptable variability

As explained in the previous section, the measurement error can cause incorrect decisions about the product. The impact on process decisions would be as follows:

- Calling a common cause a special cause
- Calling a special cause a common cause

Measurement system variability can affect the decision regarding the stability, target and variation of a process. The basic relationship between the actual and the observed process variation is:

$$\sigma^2_{obs} = \sigma^2_{actual} + \sigma^2_{msa}$$

where

σ^2_{obs} = observed process variance

σ^2_{actual} = actual process variance

σ^2_{msa} = variance of the measurement system

The capability index[9] *Cp* is defined as

$$Cp = \frac{ToleranceRange}{6\sigma}$$

The relationship between the *Cp* index of the observed process and the *Cp* indices of the actual process and the measurement system is derived by substituting the equation for *Cp* into the observed variance equation above:

$$(Cp)^{-2}_{obs} = (Cp)^{-2}_{actual} + (Cp)^{-2}_{msa}$$

Assuming the measurement system is in statistical control and on target, the actual process *Cp* can be compared graphically to the observed *Cp*.[10]

Therefore the observed process capability is a combination of the actual process capability plus the variation due to the measurement process. To reach a specific process capability goal would require factoring in the measurement variation.

For example, if the measurement system *Cp* index were 2, the actual process would require a *Cp* index greater than or equal to 1.79 in order for the calculated (observed) index to be 1.33. If the measurement system *Cp* index were itself 1.33, the process would require **no** variation at all if the final result were to be 1.33 – clearly an impossible situation.

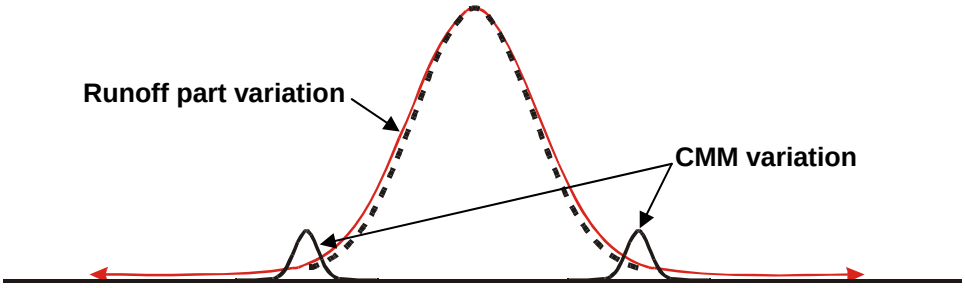
[9] Although this discussion is using *Cp*, the results hold also for the performance index *Pp*.
[10] See Appendix B for formulas and graphs.

New Process Acceptance

When a new process such as machining, manufacturing, stamping, material handling, heat treating, or assembly is purchased, there often is a series of steps that are completed as part of the buy-off activity. Oftentimes this involves some studies done on the equipment at the supplier's location and then at the customer's location.

If the measurement system used at either location is not consistent with the measurement system that will be used under normal circumstances then confusion may ensue. The most common situation involving the use of different instruments is the case where the instrument used at the supplier has higher order discrimination than the production instrument (gage). For example, parts measured with a coordinate measuring machine during buy-off and then with a height gage during production; samples measured (weighed) on an electronic scale or laboratory mechanical scale during buy-off and then on a simple mechanical scale during production.

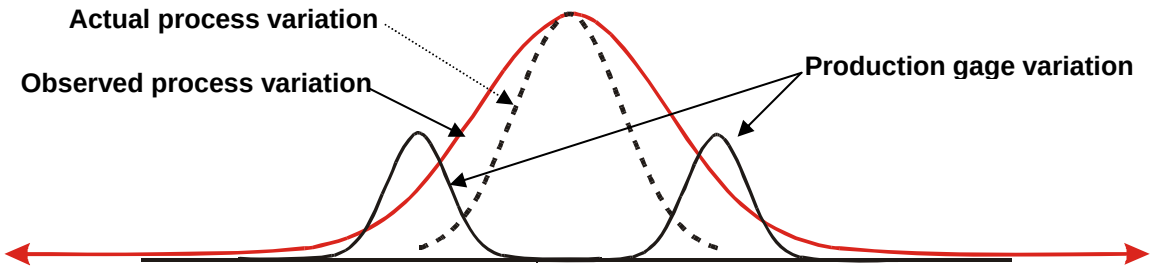
In the case where the (higher order) measurement system used during buy-off has a *GRR* of 10% and the actual process *Cp* is 2.0 the observed process *Cp* during buy-off will be 1.96.[11]



When this process is studied in production with the production gage, more variation (i.e., a smaller *Cp*) will be observed. For example, if the *GRR* of the production gage is 30% and the actual process *Cp* is still 2.0 then the observed process *Cp* will be 1.71.

A worst case scenario would be if a production gage has not been qualified but is used. If the measurement system *GRR* is actually 60% (but that fact is not known), then the observed *Cp* would be 1.28. The difference in the observed *Cp* of 1.96 versus 1.28 is due to the different measurement system. Without this knowledge, efforts may be spent, in vain, looking to see what went wrong with the new process.

[11] For this discussion, assume there is no sampling variation. In reality 1.96 will be the expected value but actual results will vary around it.



Process Setup/ Control (Funnel Experiment)

Often manufacturing operations use a single part at the beginning of the day to verify that the process is targeted. If the part measured is off target, the process is then adjusted. Later, in some cases another part is measured and again the process may be adjusted. Dr. Deming referred to this type of measurement and decision-making as *tampering*.

Consider a situation where the weight of a precious metal coating on a part is being controlled to a target of 5.00 grams. Suppose that the results from the scale used to determine the weight vary ±0.20 grams but this is not known since the measurement system analysis was never done. The operating instructions require the operator to verify the weight at setup and every hour based on one sample. If the results are beyond the interval 4.90 to 5.10 grams then the operator is to setup the process again.

At setup, suppose the process is operating at 4.95 grams but due to measurement error the operator observes 4.85 grams. According to instructions the operator attempts to adjust the process up by .15 grams. Now the process is running at 5.10 grams for a target. When the operator checks the setup this time, 5.08 grams is observed so the process is allowed to run. Over-adjustment of the process has added variation and will continue to do so.

This is one example of the funnel experiment that Dr. Deming used to describe the effects of tampering.[12] The measurement error just compounds the problem.

Four rules of the funnel experiment are:

Rule 1: Make no adjustment or take no action unless the process is unstable.

Rule 2: Adjust the process in an equal amount and in an opposite direction from where the process was last measured to be.

Rule 3: Reset the process to the target. Then adjust the process in an equal amount and in an opposite direction from the target.

Rule 4: Adjust the process to the point of the last measurement.

The setup instruction for the precious metal process is an example of Rule 3. Rules 2, 3 and 4 add progressively more variation. Rule 1 is the best choice to produce minimum variation.

[12] Deming, W. Edwards, *Out of the Crisis*, Massachusetts Institute of Technology, 1982, 1986.

Other examples of the funnel experiment are:

- Recalibration of gages based on arbitrary limits – i.e., limits not reflecting the measurement system's variability. (Rule 3)
- (Re)mastering the process control measurement system after an arbitrary number of uses without any indication or history of a change (special cause). (Rule 3)
- Autocompensation adjusts the process based on the last part produced. (Rule 2)
- On the job training (OJT) where worker A trains worker B who later trains worker C... without standard training material. Similar to the "post office" game. (Rule 4)
- Parts are measured, found to be off target, but when plotted on a control chart the process is shown to be stable – therefore, no action is taken. (Rule 1)

Section C
Measurement Strategy and Planning

Planning is key before designing and purchase of measurement equipment or systems. Many decisions made during the planning stage could affect the direction and selection of measurement equipment. What is the purpose and how will the measurement result be used? The planning stage will set the course and have a significant effect on how well the measurement process operates and can reduce possible problems and measurement error in the future.

> In some cases due to the risk involved in the component being measured or because of the cost and complexity of the measurement device, the OEM customer may use the APQP process and committee to decide on the measurement strategy at the supplier.

Not all product and process characteristics require measurement systems whose development falls under this type of scrutiny. Simple standard measurement tools like micrometers or calipers may not require this in-depth strategy and planning. A basic rule of thumb is whether the characteristic being measured on the component or sub-system has been identified in the control plan or is important in determining the acceptance of the product or process. Another guide would be the level of tolerance assigned to a specific dimension. **Common sense is the guide in any case.**

Complexity

The type, complexity, and purpose of a measurement system may drive various levels of program management, strategic planning, measurement systems analysis, or other special consideration for measurement selection, assessment and control. Simple measuring tools and devices (i.e., scales, measuring tapes, fixed-limit or attribute gages) may not require the level of management, planning, or analysis that more complex or critical measuring systems demand (i.e., master or reference, CMM, test stand, automated on-line gaging, etc.). Any measurement system may require more or less strategic planning and scrutiny depending on a given product or process situation. The decision as to the appropriate level shall be left to the APQP team assigned to the measurement process and customer. The actual degree of involvement or implementation in many of the activities below should be driven by the particular measurement system, consideration of the supporting gage control and calibration system, profound process knowledge, and common sense.

Identify the Purpose of the Measurement Process

The first step is to establish the purpose for the measurement and how the measurement will be utilized. A cross-functional team organized early in the development of the measurement process is critical in accomplishing this task. Specific considerations are made in relation to audit, process control, product and process development and analysis of the "Measurement Life Cycle".

Measurement Life Cycle

The Measurement Life Cycle concept expresses the belief that the measurement methods may change over time as one learns and improves the process. For example, measurement may start on a product characteristic to establish stability and capability of the process. This may lead to an understanding of critical process control characteristics that directly affect the part characteristics. Dependency on part characteristic information becomes less and the sampling plan may be reduced to signify this understanding (five parts per hour sample reduced to one part per shift). Also, the method of measurement may change from a CMM measurement, to some form of attribute gaging. Eventually it may be found that very little part monitoring may be required as long as the process is maintained or measuring and monitoring the maintenance and tooling may be all that is needed. The level of measurement follows the level of process understanding.

Most of the measuring and monitoring could eventually end up at suppliers of incoming material. The same measurement, on the same characteristic, at the same area of the process, over an extensive period of time is evidence of a lack of learning or a stagnant measurement process.

Criteria for a Measurement Process Design Selection

Before a measurement system can be purchased, a detailed engineering concept of the measurement process is developed. Using the purpose developed above, a cross-functional team of individuals will develop a plan and concept for the measurement system required by the design. Here are some guidelines:

The team needs to evaluate the design of the subsystem or component and identify important characteristics. These are based on customer requirements and the functionality of the subsystem or component to the total system. If the important dimensions have been identified already, evaluate the ability to measure the characteristics. For example, if the important characteristic of a plastic injection molded component was on the mold parting line, the dimensional check would be difficult and measurement variation would be high.

One method to capture issues similar to these would be to use a FMEA process to analyze areas of risk in gage design both from an ability to measure to the part to the functionality gage (Design and Process FMEA). This would aid in the development of the maintenance and calibration plan.

Develop a flow chart showing critical process steps in the manufacturing or assembly of the part or subsystem. Identify key inputs and outputs to each step in the process. This will aid in the development of the measurement equipment criteria and requirements affected by the location in the process.

A measurement plan, a list of measurement types, comes out of this investigation.[13]

For complex measurement systems, a flow chart is made of the measurement process. This would include delivery of the part or sub-system being measured, the measurement itself, and the return of the part or sub-system to the process.

Next use some method of brainstorming with the group to develop general criteria for each measurement required. One of the simple methods to use is a cause and effect diagram.[14] See the example in Figure I-B 1 as a thought starter.

A few additional questions to consider in relation to measurement planning:

- Who ought to be involved in the "needs" analysis? The flow chart and initial discussion will facilitate the identification of the key individuals.
- Why will measurement be taken and how will it be used? Will the data be used for control, sorting, qualification, etc? The way the measurement will be used can change the sensitivity level of the measurement system.
- What level of sensitivity will be required? What is the product specification? What is the expected process variability? How much of a difference between parts will the gage need to detect?
- What type of information will be provided with the gage (e.g., manuals – operating, maintenance, etc.) and what basic operator skills are required? Who will do the training?
- How are measurements taken? Will it be done manually, on a moving conveyor, off-line, automatically, etc? Are the part location and fixturing possible sources of variation? Contact or non-contact?
- How will the measurement be calibrated and will it be compared with other measurement processes? Who will be responsible for the calibration masters?
- When and where will the measurement be taken? Will the part be clean, oily, hot, etc.?

> Remember to use data to substantiate common assumptions about the measurement process. It is better to be safe and collect data on the environment, rather than to make decisions based on the wrong information and having a system developed that is not robust to environmental issues.

[13] This can be considered as a preliminary control plan.
[14] See *Guide to Quality Control*, Kaoru Ishikawa, published by Asian Productivity Organization, 1986.

Research Various Measurement Process Methods

Current measurement methods should be researched prior to investing in new equipment. Proven measurement methods may provide more reliable operation. Where possible, use measurement equipment that has a proven track record.

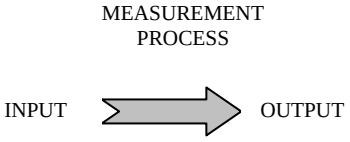
Develop and Design Concepts and Proposals

Refer to "Suggested Elements for a Measurement System Development Checklist" at the end of Chapter I, Section D, when developing and designing concepts and proposals.

During and after the fabrication of the measurement equipment and development of the measurement process (methods, training, documentation, etc.), experimental studies and data collection activities will be performed. These studies and data will be used to understand this measurement process so that this process and future processes may be improved.

Section D
Measurement Source Development

This section addresses the quotation/procurement timeframe of the life of a measurement process. It has been constructed to be a self-contained discussion about the process of developing a measurement process quotation package, obtaining responses to that package, awarding the project, completing final design, developing the measurement process, and, finally, marrying that measurement process to the production process for which it was created. It is strongly encouraged that this chapter not be used without reading and understanding the entire discussion about a measurement process. To obtain the most benefit from the measurement process, study and address it **as a process** with inputs and outputs.[15]



This chapter was written with the team philosophy in mind. It is not a job description for the buyer or purchasing agent. The activities described here will require team involvement to be completed successfully and it should be administered within the overall framework of an Advanced Product Quality Planning (APQP) team. This can result in healthy interplay between various team functions – concepts arising out of the planning process may be modified before the gage supplier arrives at a final design that satisfies the measurement system requirements.

Generally, the "acquisition process" begins with formal communication between the customer and supplier for a given project. Up-front communication is crucial to the success of the project, since the groundwork necessary for an effective future customer/supplier relationship will be done at this stage. The acquisition process begins with the customer's formal presentation of the intent of the project in the form of a Request For Quote (RFQ) followed by the supplier's formal explanation of their proposal to meet this intent (the Quotation). The customer and supplier(s) need to thoroughly understand the project requirements, what the deliverables will be and the methods by which both are to be achieved. This understanding is derived from accurate timely communication between the two parties.

Once a concept has been agreed upon and a customer/supplier relationship has been established for the project at hand, the detailed design, fabrication of the measurement process, and development activities can commence. Communication between the customer and the supplier at this time is especially important. Since there may be several levels of concept approvals to be carried out, and possible environmental changes and the potential of team members changing, the measurement process project could falter or even fail. This risk will be reduced if frequent, detailed communication is maintained and documented between the customer and supplier and formal responsibility (an individual) for maintaining communication is designated

[15] See Chapter I, Section B

by both parties. The ideal forum and format for this activity is the Advanced Product Quality Planning (APQP) process.

After the measurement process has been conceptually designed, the activities surrounding the acquisition of the process/system can begin.

Datum Coordination

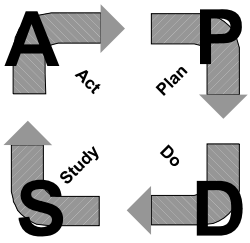
Ideally, with the current prevalence in the use of Geometric Dimensioning & Tolerancing (GD&T), datums need to be coordinated (i.e., made identical) throughout the manufacturing process and the measurement system and this needs to be established very early in the APQP process. Initial responsibility for this may lie with the product design engineer, dimensional control, etc. depending on the specific organization. When datum schemes do not match throughout a manufacturing process, particularly in the measurement systems, this leads to a situation where the wrong things may be measured, and there may be fit problems, etc., leading to ineffective control of the manufacturing process.

There may be times when a datum scheme used in a final assembly cannot possibly match that used in a sub-component manufacturing process. When such is the case, it can be established as early as possible in the APQP process so that all team members understand possible difficulties and conflicts that may lie ahead and have every opportunity to do something about it. During this process, different datum schemes may need to be explored in order to understand the impact of these differences.

Certain commodities present features which can yield more problems than others, such as camshaft centering, or other round, cylindrical or tubular characteristics. For example, a camshaft must be manufactured on centers but the important product features are in its lobes. One method or datum scheme may be required for manufacturing whereas another scheme is required for measurement of the final product measurement.

Prerequisites and Assumptions

Before discussing the development of a gage supplier, it will be assumed that issues such as "correct" engineering product design (GD&T) and "correct" process design (one which allows for measurement at the proper time and location in the process) have been resolved. However this should not detract from consideration of these issues with appropriate team members early in the APQP process.



It is assumed that the gage supplier will be involved with the APQP process, a *team* approach. The gage supplier will develop a clear appreciation of the overall production process and product usage so that his role is understood not only by him but by others on the team (manufacturing, quality, engineering, etc.).

There may be slight overlap in some activities or the order of those activities depending on the particular program/project or other constraints. For instance, the APQP team without much input from a gage source may develop certain gage concepts. Other concepts may require the expertise of the gage source. This may be driven by the complexity of the measurement system and a team decision as to what makes sense.

Gage Source Selection Process

Develop the Quotation Package

Detailed Engineering Concept

Before a measurement process request for quotation package can be supplied to a potential supplier for formal proposals, a detailed engineering concept of the measurement process needs to be developed. The team of individuals that will employ and be responsible for the maintenance and continual improvement of the measurement process have direct responsibility for developing the detailed concept. This can be part of the APQP team. To better develop this concept, several questions need to be answered.

The team may research various issues to help decide which direction or path will be followed for designing the measurement process. Some may be dictated or heavily implied by the product design. Examples of the multitude of possible issues that need to be addressed by the team when developing this detailed concept may be found in the "Suggested Elements for a Measurement System Development Checklist" at the end of this section.

> All too often, customers rely too heavily on suppliers for solutions. Before a customer asks a supplier to suggest solutions to process problems, the foundation and intent of the process needs to be thoroughly understood and anticipated by the team that owns that process. Then and only then will the process be properly used, supported and improved upon.

Preventive Maintenance Considerations

What activities should be scheduled for preventive maintenance (e.g., lubrication, vibration analysis, probe integrity, parts replacement, etc.)? Much of these activities will depend on the complexity of the measurement system, device or apparatus. Simpler gages may require only an inspection at regular intervals, whereas more complex systems may require ongoing detailed statistical analyses and a team of engineers to maintain in a predictive fashion.

Planning preventive maintenance activities should coincide with the initiation of the measurement process planning. Many activities, such as draining air filters daily, lubricating bearings after the designated number of operating hours, etc., can be planned before the measurement system is completely built, developed and implemented. In fact this is preferable and improves advanced measurement planning and costs. Data collection methods and maintenance recommendations related to these activities can be obtained from the original manufacturer, or developed by plant engineering, manufacturing and quality personnel. After the measurement process is implemented and in use, data pertaining to the function of the measurement process need to be collected and plotted over time. Simple analytical methods (run charts, trend analysis) can be conducted to determine the stability of the system. Eventually, as the judgment of system stability dictates, preventive maintenance routines can be scheduled accordingly. Conducting preventive maintenance on a stable system, based on time series information, will be less wasteful than conducting preventive maintenance on a system with traditional techniques.

Specifications

Specifications serve as guidelines for both the customer and supplier in the design and build process. These guidelines serve to communicate acceptable standards. Acceptable standards may be considered in two categories:

- Design Standards
- Build Standards

Format of the design standards may be different depending on who is paying for the project. Cost issues may affect the format. Generally, it is a good idea to have sufficient documented design detail that the design may be built or repaired to original intent by any qualified builder – however, this decision

may be driven by cost and criticality. The required format of the final design may be some form of computer assisted design (CAD) or hardcopy engineering drawings. It may involve engineering standards chosen from those of the OEM, SAE, ASTM, or other organization, and the gage supplier must have access to the latest level and understand these standards. The OEM may require the use of particular standards at either the design or build phase and may even require formal approvals before the measurement system may be released for use.

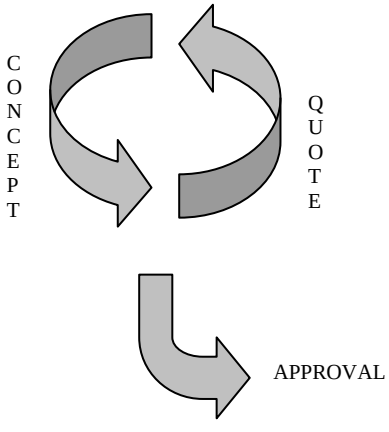
Design standards will detail the method of communicating the design (CAD – e.g., CATIA, Unigraphics, IGES, manual hardcopy, etc.) to the builder. It may also cover performance standards for a more complex measurement system.

Build standards will cover the tolerances to which the measurement system must be built. Build tolerance should be based on a combination of the capabilities of the process used to produce the gage or gage component, and the criticality of the intended measurement. Build tolerance should not be a mere given percent of product tolerance alone.

> If duplicate fixtures or systems are required, proper planning and standardizing can lead to interchangeability and flexibility.

Use of standard(ized) components or subassemblies also leads to interchangeability, flexibility, reduced cost and, generally, less long-term measurement error.

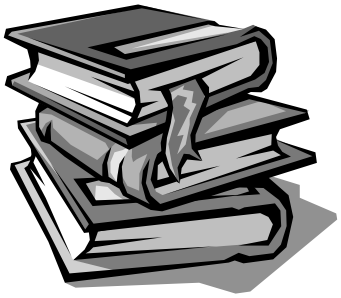
Evaluate the Quotations



As quotations are received, the team ought to assemble to review and evaluate them. Certain items can be noted:

- ✓ Are the basic requirements met?
- ✓ Are there any outstanding concerns?
- ✓ Do any of the suppliers exhibit an exceptional condition and why? (*An exceptional condition could be a significant disparity with regard to price or delivery – this would not necessarily be discounted as a negative factor – one supplier may have discovered an item that others overlooked*.)
- ✓ Do the concepts promote simplicity and maintainability?

Documentation is sometimes overlooked when acquiring a measurement process. The significance that documentation takes with any successful project is often misunderstood. The usual strategy behind documentation is to provide an original set of mechanical and electrical designs (CAD or hardcopy drawings) for the measurement process hardware at the time of delivery. This may satisfy initial implementation requirements, but this documentation does nothing with regard to defining potential wear points, suggesting possible trouble areas or describing how to use the process. Thus,



the required documentation for any process ought to include more than assembly and detailed drawings of the measurement equipment.

Effective documentation for any system serves the same purpose as a good map on a trip. For example, it will suggest to the user how to get from one point to another (user instructions or gage instructions). It provides the user with possible alternative routes to reach the desired destinations (troubleshooting guides or diagnostic trees) if the main route is blocked or closed.

A complete documentation package may include:

- Reproducible set of assembly and detailed mechanical drawings (CAD or hardcopy) (including any required masters)
- Reproducible set of electrical hard-wiring, logic and software
- Suggested spare parts list of heavy use or wear items/details. This list should include items that may require considerable lead-time to acquire
- Maintenance manuals with machine drawing cutaways and steps to properly assemble and disassemble machine components
- Manuals defining utility requirements for setup and operation and machine transport requirements (e.g., load bearing members)
- Diagnostic trees and a troubleshooting guide
- Certification reports (traceable to NIST where applicable)
- Calibration instructions
- User manuals that can be used by the technical support personnel, the system operator and maintenance personnel

The above list can be used as a checklist when organizing the quotation package; however it is not necessarily all-inclusive.

The central theme here is communication. Since documentation is a form of communication, the team and others ought to be involved at every level of the development of the measurement process documentation package.

Qualification at the Supplier

The gage or measurement system should be given a full dimensional layout and functional test, where applicable, at the measurement system supplier before shipment. Obviously, the chosen supplier must have qualified measurement equipment and personnel on site in order to accomplish this. If not, pre-arrangements should have been made to have this work done at an outside independent qualified laboratory. Results of such dimensional layout and/or testing should be done in accordance with customer design and build standards and be fully documented and available for customer review.

After successful dimensional layout, the supplier should perform a preliminary but formal measurement systems analysis. This again pre-requires that the supplier have the personnel, knowledge and experience to accomplish the appropriate analysis. The customer should predetermine with the supplier (and perhaps the OEM) exactly what sort of analysis is required at this point and should be aware of any guidance the supplier might need. Some issues that may need discussion, negotiation or common agreement are:

- Objective of the preliminary MSA study :
  - ✓ Gage repeatability (GR[16]) versus gage repeatability and reproducibility (*GRR*)
  - ✓ Assessment of bias and/or linearity
  - ✓ Assessment of the customer purpose for measurement
- Quantity of pieces, trials and operators in study
  - ✓ Acceptance criteria
- Use of supplier personnel vs. customer supplied personnel
- Necessary training for personnel
  - ✓ Are they qualified?
  - ✓ Do they understand intent?
  - ✓ What software might be used?Whatever results are achieved at this point in time, it should be realized that these are merely preliminary and *judgment* may be needed as to the acceptability of the results.

Shipment

CHECKLIST

- When should the equipment be shipped?
- How should it be shipped?
- Who removes equipment from the truck or rail car?
- Is insurance required?
- Should documentation be shipped with the hardware?
- Does the customer have the proper equipment to unload the hardware?
- Where will the system be stored until shipment?

[16] See Appendix D.

- Where will the system be stored until implementation?
- Is the shipping documentation complete and easily understandable for the loader, transporter, unloader and installation crew?

Qualification at the Customer

Generally, what was done to qualify the measurement system above at the supplier before shipment should be repeated in some manner at the customer once delivery is completed. Since this becomes the first real opportunity to study the measurement system in its intended environment, acceptance standards and analysis methods used here should be considered seriously. Attention to detail on the part of all parties involved is paramount to the eventual success of this measurement system and the use of the data it generates.

Before any measurement analysis is begun after receipt, the measurement system should undergo a full dimensional layout to confirm it meets build requirements/standards. The extent of this layout may be balanced against layout work done previously at the measurement system supplier before shipment and confidence in the quality of the layout results done at the supplier as well as the lack of potential shipping damage. When comparing results before and after shipment, be aware that there will likely be some differences in these measurements because of differences in these measurement systems.

Documentation Delivery

The information that is required, at a minimum, to aid implementation and startup of any system is the following: (This information ought to be delivered to the customer prior to delivery.)

- CAD or hardcopy drawings, if required by team
- Process flow diagram of the system, where applicable
- User manuals
  - ✓ Maintenance/service manual
  - ✓ Spare parts list
  - ✓ Troubleshooting guide
- Calibration instructions
- Any special considerations

At the outset, the delivered documentation needs to be noted as preliminary. Original or reproducible documentation does not need to be delivered at this time because potential revision may be necessary after implementation. In fact, it is a wise idea to not have the original documentation package delivered until after the entire system is implemented – suppliers are generally more efficient with updating documentation than customers.

Suggested Elements for a Measurement System Development Checklist

This list should be modified based on the situation and type of measurement system. The development of the final checklist should be the result of collaboration between the customer and supplier.

Measurement System Design and Development Issues:

- **What is to be measured?** What type of characteristic is it? Is it a mechanical property? Is it dynamic or stationary? Is it an electrical property? Is there significant within-part variation?
- **For what purpose will the results (output) of the measurement process be used?** Production improvement, production monitoring, laboratory studies, process audits, shipping inspection, receiving inspection, responses to a D.O.E.?
- **Who will use the process?** Operators, engineers, technicians, inspectors, auditors?
- **Training required**: Operator, maintenance personnel, engineers; classroom, practical application, OJT, apprenticeship period.
- **Have the sources of variation been identified?** Build an error model (S.W.I.P.E. or P.I.S.M.O.E.A.) using teams, brainstorming, profound process knowledge, cause & effect diagram or matrix.
- **Has a FMEA been developed for the measurement system?**
- **Flexible vs. dedicated measurement systems:** Measurement systems can either be permanent and dedicated or they can be flexible and have the ability to measure different types of parts; e.g., doghouse gages, fixture gaging, coordinate measurement machine, etc. Flexible gaging will be more expensive, but can save money in the long run.
- **Contact vs. non-contact:** Reliability, type of feature, sample plan, cost, maintenance, calibration, personnel skill required, compatibility, environment, pace, probe types, part deflection, image processing . This may be determined by the control plan requirements and the frequency of the measurement (Full contact gaging may get excessive wear during continuous sampling). Full surface contact probes, probe type, air feedback jets, image processing, CMM vs. optical comparator, etc.
- **Environment:** Dirt, moisture, humidity, temperature, vibration, noise, electro-magnetic interference (EMI), ambient air movement, air contaminants, etc. Laboratory, shop floor, office, etc? Environment becomes a key issue with low, tight tolerances in the micron level. Also, in cases that CMM, vision systems, ultrasonic, etc. This could be a factor in auto-feedback in-process type measurements. Cutting oils, cutting debris, and extreme temperatures could also become issues. Is a clean room required?
- **Measurement and location points:** Clearly define, using GD&T, the location of fixturing and clamping points and where on the part the measurements will be taken.
- **Fixturing method:** Free state versus clamped part holding.
- **Part orientation:** Body position versus other.
- **Part preparation:** Should the part be clean, non-oily, temperature stabilized, etc. before measurement?
- **Transducer location**: Angular orientation, distance from primary locators or nets.

- **Correlation issue #1 – duplicate gaging:** Are duplicate (or more) gages required within or between plants to support requirements? Building considerations, measurement error considerations, maintenance considerations. Which is considered *the* standard? How will each be qualified?

- **Correlations issue #2 – methods divergence:** Measurement variation resulting from different measurement system designs performing on the same product/process within accepted practice and operation limits (e.g., CMM versus manual or open-setup measurement results).

- **Automated vs. manual:** on-line, off-line, operator dependencies.

- **Destructive versus nondestructive measurement (NDT):** Examples: **t**ensile test, salt spray testing, plating/paint coating thickness, hardness, dimensional measurement, image processing, chemical analysis, stress, durability, impact, torsion, torque, weld strength, electrical properties, etc.

- **Potential measurement range:** size and expected range of conceivable measurements.

- **Effective resolution:** Is measurement sensitive to physical change (ability to detect process or product variation) for a particular application acceptable for the application?

- **Sensitivity:** Is the size of the smallest input signal that results in a detectable (discernable) output signal for this measurement device acceptable for the application? Sensitivity is determined by inherent gage design and quality (OEM), in-service maintenance, and operating condition.

Measurement System Build Issues (equipment, standard, instrument):

- **Have the sources of variation identified in the system design been addressed?** Design review; verify and validate.

- **Calibration and control system:** Recommended calibration schedule and audit of equipment and documentation. Frequency, internal or external, parameters, in-process verification checks.

- **Input requirements:** Mechanical, electrical, hydraulic, pneumatic, surge suppressors, dryers, filters, setup and operation issues, isolation, discrimination and sensitivity.

- **Output requirements:** Analog or digital, documentation and records, file, storage, retrieval, backup.

- **Cost:** Budget factors for development, purchase, installation, operation and training.

- **Preventive maintenance:** Type, schedule, cost, personnel, training, documentation.

- **Serviceability:** Internal and external, location, support level, response time, availability of service parts, standard parts list.

- **Ergonomics:** Ability to load and operate the machine without injuries over time. Measurement device discussions need to focus on issues of how the measurement system is interdependent with the operator.

- **Safety considerations:** Personnel, operation, environmental, lock-out.

- **Storage and location:** Establish the requirements around the storage and location of the measurement equipment. Enclosures, environment, security, availability (proximity) issues.

- **Measurement cycle time:** How long will it take to measure one part or characteristic? Measurement cycle integrated to process and product control.

- **Will there be any disruption to process flow, lot integrity, to capture, measure and return the part?**

- **Material handling:** Are special racks, holding fixtures, transport equipment or other material handling equipment needed to deal with parts to be measured or the measurement system itself?

- **Environmental issues:** Are there any special environmental requirements, conditions, limitations, either affecting this measurement process or neighboring processes? Is special exhausting required? Is temperature or humidity control necessary? Humidity, vibration, noise, EMI, cleanliness.

- **Are there any special reliability requirements or considerations***?* Will the equipment hold up over time? Does this need to be verified ahead of production use?

- **Spare parts:** Common list, adequate supply and ordering system in place, availability, lead-times understood and accounted for. Is adequate and secure storage available? (bearings, hoses, belts, switches, solenoids, valves, etc.)

- **User instructions:** Clamping sequence, cleaning procedures, data interpretation, graphics, visual aids, comprehensive. Available, appropriately displayed.

- **Documentation:** Engineering drawings, diagnostic trees, user manuals, language, etc.

- **Calibration:** Comparison to acceptable standards. Availability and cost of acceptable standards. Recommended frequency, training requirements. Down-time required?

- **Storage:** Are there any special requirements or considerations regarding the storage of the measurement device? Enclosures, environment, security from damage/theft, etc.

- **Error/Mistake proofing:** Can known measurement procedure mistakes be corrected easily (too easily?) by the user? Data entry, misuse of equipment, error proofing, mistake proofing.

Measurement System Implementation Issues (process):

- **Support:** Who will support the measurement process*?* Lab technicians, engineers, production, maintenance, outside contracted service?

- **Training:** What training will be needed for operators/inspectors/technicians/engineers to use and maintain this measurement process? Timing, resource and cost issues. Who will train? Where will training be held? Lead-time requirements? Coordinated with actual use of measurement process.

- **Data management:** How will data output from this measurement process be managed? Manual, computerized, summary methods, summary frequency, review methods, review frequency, customer requirements, internal requirements. Availability, storage, retrieval, backup, security. Data interpretation.

- **Personnel:** Will personnel need to be hired to support this measurement process? Cost, timing, availability issues. Current or new.

- **Improvement methods:** Who will improve the measurement process over time*?* Engineers, production, maintenance, quality personnel? What evaluation methods will be used? Is there a system to identify needed improvements?

- **Long-term stability:** Assessment methods, format, frequency, and need for long-term studies. Drift, wear, contamination, operational integrity. Can this long-term error be measured, controlled, understood, predicted?

- **Special considerations:** Inspector attributes, physical limitations or health issues: colorblindness, vision, strength, fatigue, stamina, ergonomics.

Section E
Measurement Issues

Three fundamental issues must be addressed when evaluating a measurement system:

1) The measurement system must demonstrate adequate sensitivity.
   - ✓ First, does the instrument (and standard) have adequate discrimination? Discrimination (or class) is fixed by design and serves as the basic starting point for selecting a measurement system. Typically, the Rule of Tens has been applied, which states that instrument discrimination should divide the tolerance (or process variation) into ten parts or more.
   - ✓ Second, does the measurement system demonstrate effective resolution? Related to discrimination, determine if the measurement system has the sensitivity to detect changes in product or process variation for the application and conditions.

2) The measurement system must be stable.
   - ✓ Under repeatability conditions, the measurement system variation is due to common causes only and not special (chaotic) causes.
   - ✓ The measurement analyst must always consider practical and statistical significance.

3) The statistical properties (errors) are consistent over the expected range and adequate for the purpose of measurement (product control or process control).

The long-standing tradition of reporting measurement error only as a percent of tolerance is inadequate for the challenges of the marketplace that emphasize strategic and continuous process improvement. As processes change and improve, a measurement system must be re-evaluated for its intended purpose. It is essential for the organization (management, measurement planner, production operator, and quality analyst) to understand the purpose of measurement and apply the appropriate evaluation.

Types of Measurement System Variation

It is often assumed that measurements are exact, and frequently the analysis and conclusions are based upon this assumption. An individual may fail to realize there is variation in the measurement system which affects the individual measurements, and subsequently, the decisions based upon the data. Measurement system error can be classified into five categories: bias, repeatability, reproducibility, stability and linearity.

One of the objectives of a measurement system study is to obtain information relative to the amount and types of measurement variation associated with a measurement system when it interacts with its environment. This information is valuable, since for the average production process, it is far more practical to recognize repeatability and calibration bias and establish reasonable limits for these, than to provide extremely accurate gages with very high repeatability. Applications of such a study provide the following:

- A criterion to accept new measuring equipment
- A comparison of one measuring device against another
- A basis for evaluating a gage suspected of being deficient
- A comparison for measuring equipment before and after repair
- A required component for calculating process variation, and the acceptability level for a production process
- Information necessary to develop a Gage Performance Curve (*GPC*),[17] which indicates the probability of accepting a part of some true value

The following definitions help describe the types of error or variation associated with a measurement system, so that each term is clearly understood for subsequent discussion. An illustration is given for each definition which graphically displays the meaning of each term.

[17] See Chapter V, Section C.

Definitions and Potential Sources of Variation

Operational Definition

"An operational definition is one that people can do business with. An operational definition of safe, round, reliable, or any other quality [characteristic] must be communicable, with the same meaning to vendor as to the purchaser, same meaning yesterday and today to the production worker. Example:

1) A specific test of a piece of material or an assembly
2) A criterion (or criteria) for judgment
3) Decision: yes or no, the object or the material did or did not meet the criterion (or criteria)" FP18

Standard

A standard is anything taken by general consent as a basis for comparison; an accepted model. It can be an *artifact* or *ensemble* (instruments, procedures, etc.) set up and established by an authority as a rule for the measure of quantity, weight, extent, value or quality.

The concept of ensemble was formalized in ANSI/ASQC Standard M1-1996.[19] This term was used to stress the fact that all of the influences affecting the measurement uncertainty need to be taken into account; e.g., environment, procedures, personnel, etc. "An example of a simple ensemble would be an ensemble for the calibration of gage blocks consisting of a standard gage block, a comparator, an operator, environment, and the calibration procedure."

Reference Standards

A standard, generally of the highest metrological quality available at a given location, from which measurements made at that location are derived.

Measurement and Test Equipment (M&TE)

All of the measurement instruments, measurement standards, reference materials, and auxiliary apparatus that are necessary to perform a measurement.

Calibration Standard

A standard that serves as a reference in the performance of routine calibrations. Intended to act as a buffer between the calibration workload and the laboratory's reference standard(s).

[18] W. E. Deming, *Out of the Crisis* (1982, 1986), p. 277.

[19] This definition was later updated as Measurement and Test Equipment or M&TE by subsequent military standards.

Transfer Standard

A standard used to compare a separate standard of known value to the unit being calibrated.

Master

A standard used as a reference in a calibration process. May also be termed as reference or calibration standard.

Working Standard

A standard whose intended use is to perform routine measurements within the laboratory, not intended as a calibration standard, but may be utilized as a transfer standard.

> Careful consideration needs to be given to the material(s) selected for a standard. The materials employed ought to reflect the use and scope of the measurement system, as well as time-based sources of variation such as wear and environmental factors (temperature, humidity, etc.).

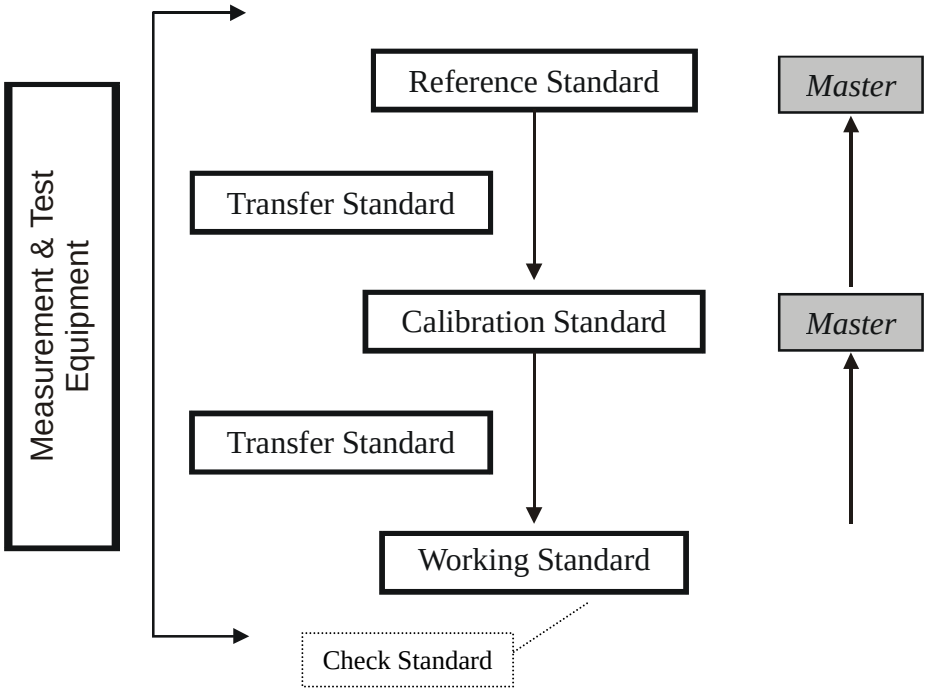


Figure I-E 1: Relationships among the Various Standards

Check Standard

A measurement artifact that closely resembles what the process is designed to measure, but is inherently more stable than the measurement process being evaluated.

Reference Value

A reference value, also known as the accepted reference value or master value, is a value of an artifact or ensemble that serves as an agreed upon reference for comparison. Accepted reference values are based upon the following:

- Determined by averaging several measurements with a higher level (e.g., metrology lab or layout equipment) of measuring equipment
- Legal values: defined and mandated by law
- Theoretical values: based on scientific principles
- Assigned values: based on experimental work (supported by sound theory) of some national or international organization
- Consensus values: based on collaborative experimental work under the auspices of a scientific or engineering group; defined by a consensus of users such as professional and trade organizations
- Agreement values: values expressly agreed upon by the affected parties

In all cases, the reference value needs to be based upon an operational definition and the results of an acceptable measurement system. To achieve this, the measuring system used to determine the reference value should include:

- Instrument(s) with a higher order discrimination and a lower measurement system error than the systems used for normal evaluation
- Be calibrated with standards traceable to the NIST or other NMI

True Value

The true value is the "actual" measure of the part. Although this value is unknown and unknowable, it is the target of the measurement process. Any individual reading ought to be as close to this value as (economically) possible. Unfortunately, the true value can never be known with certainty. The reference value is used as the best approximation of the true value in all analyses. Because the reference value is used as a surrogate for the true value, these terms are commonly used interchangeably. This usage is not recommended.[20]

[20] See also ASTM E177-90a.

Discrimination

Discrimination is the amount of change from a reference value that an instrument can detect and faithfully indicate. This is also referred to as readability or resolution.

The measure of this ability is typically the value of the smallest graduation on the scale of the instrument. If the instrument has "coarse" graduations, then a half-graduation can be used.

A general rule of thumb is the measuring instrument discrimination ought to be at least one-tenth of the range to be measured. Traditionally this range has been taken to be the product specification. Recently the 10 to 1 rule is being interpreted to mean that the measuring equipment is able to discriminate to at least one-tenth of the process variation. This is consistent with the philosophy of continual improvement (i.e., the process focus is a customer designated target).

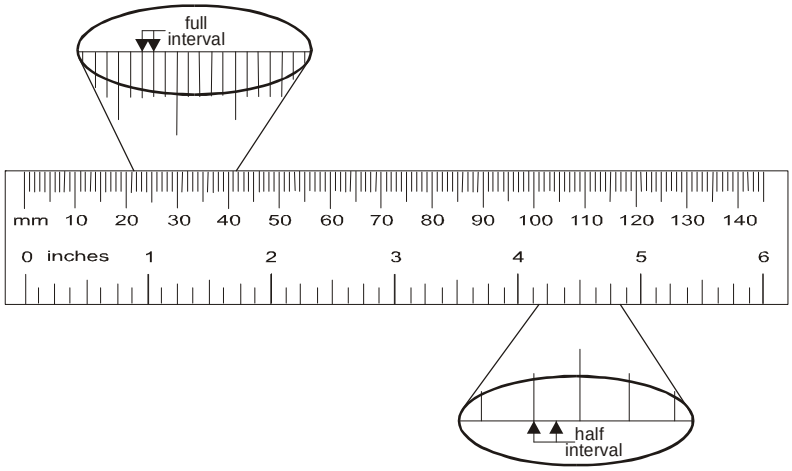


Figure I-E 2: Discrimination

The above rule of thumb can be considered as a starting point to determine the discrimination since it does not include any other element of the measurement system's variability.

Because of economic and physical limitations, the measurement system will not perceive all parts of a process distribution as having separate or different measured characteristics. Instead the measured characteristic will be grouped by the measured values into data categories. All parts in the same data category will have the same value for the measured characteristic.

If the measurement system lacks discrimination (sensitivity or effective resolution), it may not be an appropriate system to identify the process variation or quantify individual part characteristic values. If that is the case, better measurement techniques should be used.

The discrimination is unacceptable for analysis if it cannot detect the variation of the process, and unacceptable for control if it cannot detect the special cause variation (See Figure I-E 3).

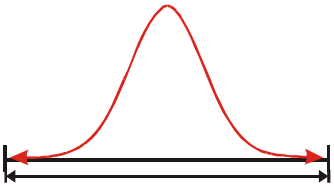
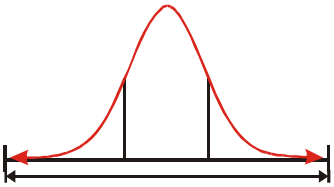
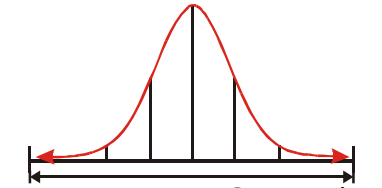
Number of Categories	Control	Analysis
 1 Data Category	Can be used for control only if: • The process variation is small when compared to the specifications • The loss function is flat over the expected process variation • The main source of variation causes a mean shift	• ***Unacceptable*** for estimating process parameters and indices • Only indicates whether the process is producing conforming or nonconforming parts
 2 - 4 Data Categories	• Can be used with semi-variable control techniques based on the process distribution • Can produce insensitive variables control charts	• Generally unacceptable for estimating process parameters and indices since it only provides coarse estimates
 5 or more Data Categories	• Can be used with variables control charts	• ***Recommended***

Figure I-E 3: Impact of Number of Distinct Categories (*ndc*) of the Process Distribution on Control and Analysis Activities

Symptoms of inadequate discrimination may appear in the range chart. Figure I-E 4 contains two sets of control charts derived from the same data. Control Chart (a) shows the original measurement to the nearest thousandth of an inch. Control Chart (b) shows these data rounded off to the nearest hundredth of an inch. Control Chart (b) appears to be out of control due to the artificially tight limits. The zero ranges are more a product of the rounding off than they are an indication of the subgroup variation.

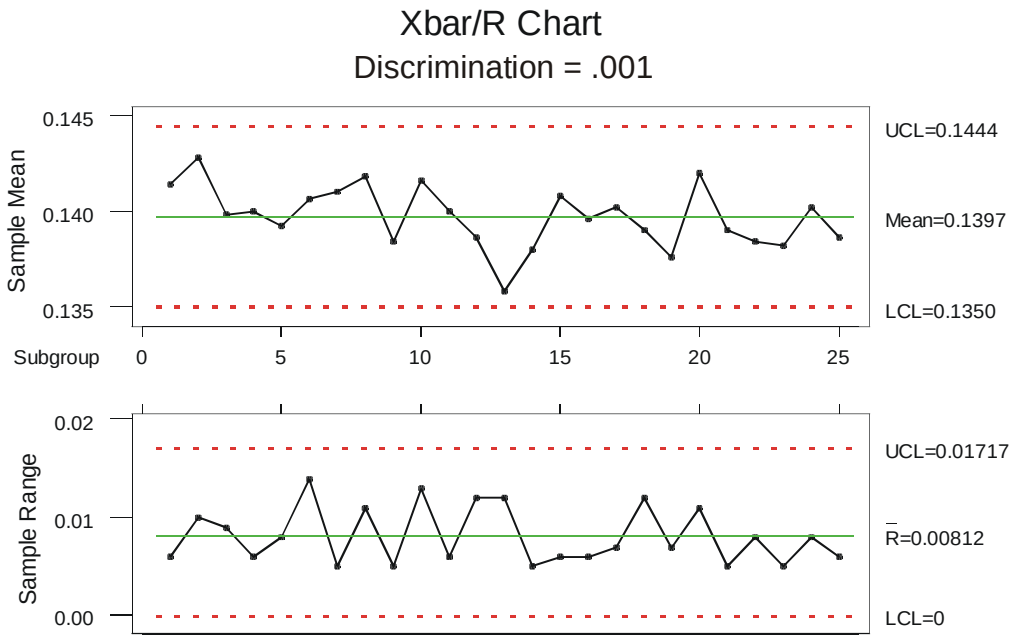
A good indication of inadequate discrimination can be seen on the SPC range chart for process variation. In particular, when the range chart shows only one, two, or three possible values for the range within the control limits, the measurements are being made with inadequate discrimination. Also, if the range chart shows four possible values for the range within control limits and more than one-fourth of the ranges are zero, then the measurements are being made with inadequate discrimination. Another good indication of inadequate discrimination is on a normal probability plot where the data will be stacked into buckets instead of flowing along the 45 degree line.

Returning to Figure I-E 4, Control Chart (b), there are only two possible values for the range within the control limits (values of 0.00 and 0.01). Therefore, the rule correctly identifies the reason for the lack of control as inadequate discrimination (sensitivity or effective resolution).

This problem can be remedied, of course, by changing the ability to detect the variation within the subgroups by increasing the discrimination of the measurements. A measurement system will have adequate discrimination if its apparent resolution is small relative to the process variation. Thus a recommendation for adequate discrimination would be for the apparent resolution to be at most one-tenth of total process six sigma standard deviation instead of the traditional rule which is the apparent resolution be at most one-tenth of the tolerance spread.

Eventually, there are situations that reach a stable, highly capable process using a stable, “best-in-class” measurement system at the practical limits of technology. Effective resolution may be inadequate and further improvement of the measurement system becomes impractical. In these special cases, measurement planning may require alternative process monitoring techniques. Customer approval will typically be required for the alternative process monitoring technique.

A



B

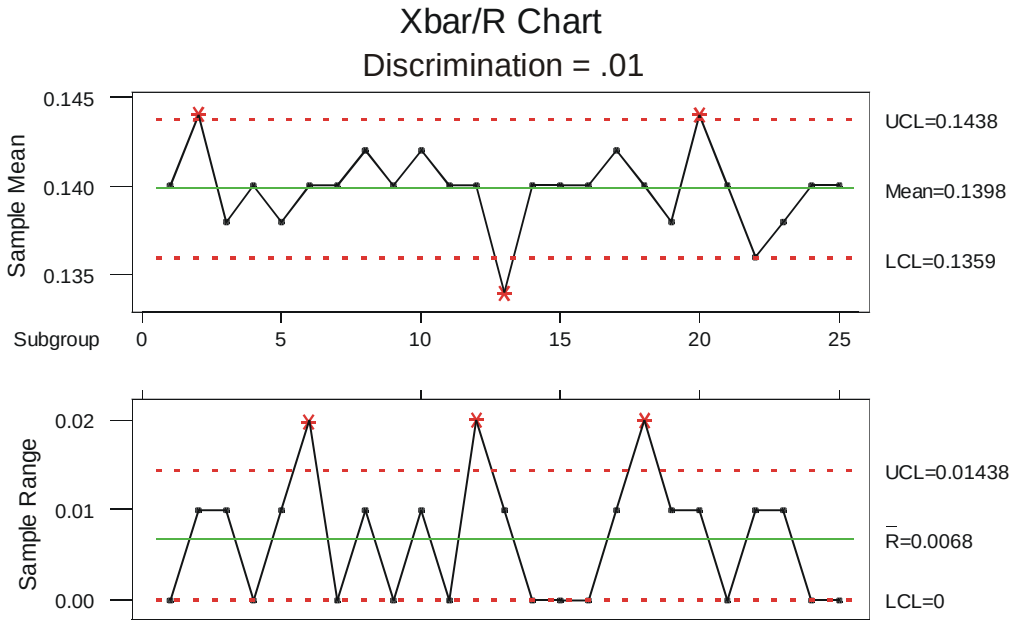


Figure I-E 4: Process Control Charts[21]

[21] Figure I-E 4 was developed using data from *Evaluating The Measurement Process*, by Wheeler and Lyday, Copyright 1989, SPC Press, Inc., Knoxville, Tennessee.

Measurement Process Variation

For most measurement processes, the total measurement variation is usually described as a normal distribution. Normal probability is an assumption of the standard methods of measurement systems analysis. In fact, there are measurement systems that are not normally distributed. When this happens, and normality is assumed, the MSA method may overestimate the measurement system error. The measurement analyst must recognize and correct evaluations for non-normal measurement systems.

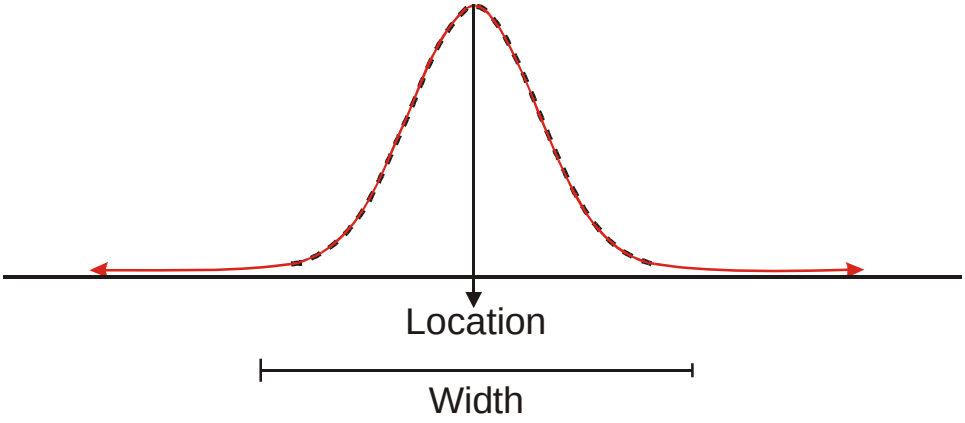


Figure I-E 5: Characteristics of the Measurement Process Variation

Location Variation

Accuracy

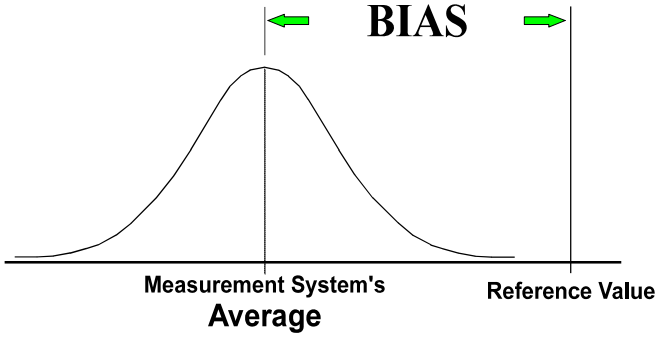
Accuracy is a generic concept of exactness related to the closeness of agreement between the average of one or more measured results and a reference value. The measurement process must be in a state of statistical control, otherwise the accuracy of the process has no meaning.

In some organizations accuracy is used interchangeably with bias. The ISO (International Organization for Standardization) and the ASTM (American Society for Testing and Materials) use the term accuracy to embrace both bias and repeatability. In order to avoid confusion which could result from using the word *accuracy*, ASTM recommends that only the term *bias* be used as the descriptor of location error. This policy will be followed in this text.

Bias

Bias is often referred to as “accuracy.” Because “accuracy” has several meanings in literature, its use as an alternate for “bias” is not recommended.

Bias is the difference between the true value (reference value) and the observed average of measurements on the same characteristic on the same part. Bias is the measure of the systematic error of the measurement system.



It is the contribution to the total error comprised of the combined effects of all sources of variation, known or unknown, whose contributions to the total error tends to offset consistently and predictably all results of repeated applications of the same measurement process at the time of the measurements.

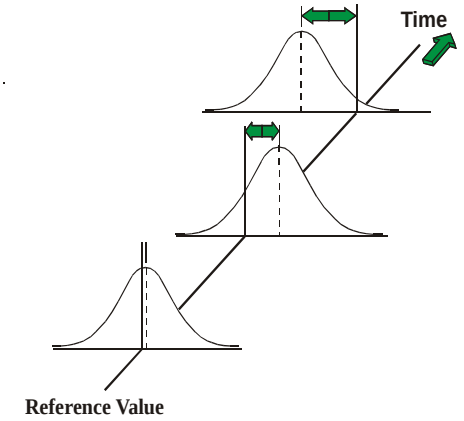
Possible causes for excessive bias are:

- Instrument needs calibration
- Worn instrument, equipment or fixture
- Worn or damaged master, error in master
- Improper calibration or use of the setting master
- Poor quality instrument – design or conformance
- Linearity error
- Wrong gage for the application
- Different measurement method – setup, loading, clamping, technique
- Measuring the wrong characteristic
- Distortion (gage or part)
- Environment – temperature, humidity, vibration, cleanliness
- Violation of an assumption, error in an applied constant
- Application – part size, position, operator skill, fatigue, observation error (readability, parallax)

The measurement procedure employed in the calibration process (i.e., using "masters") should be as identical as possible to the normal operation's measurement procedure.

Stability

Stability (or drift) is the total variation in the measurements obtained with a measurement system on the same master or parts when measuring a single characteristic over an **extended time period**. That is, stability is the change in bias over time.

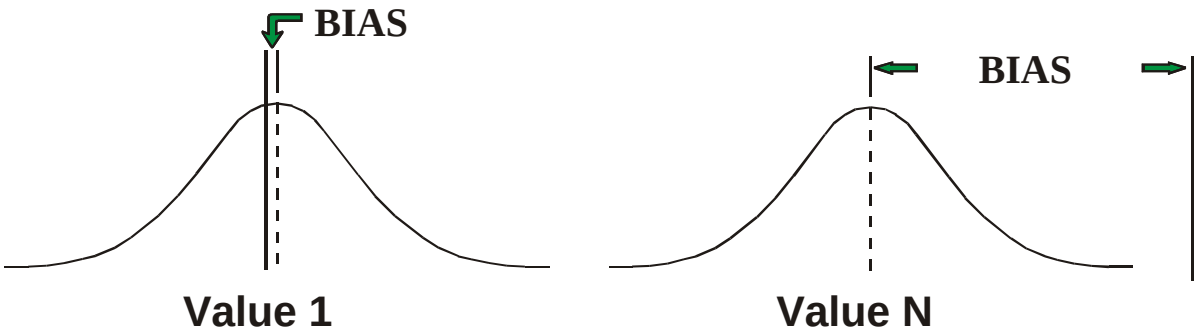


Possible causes for instability include:

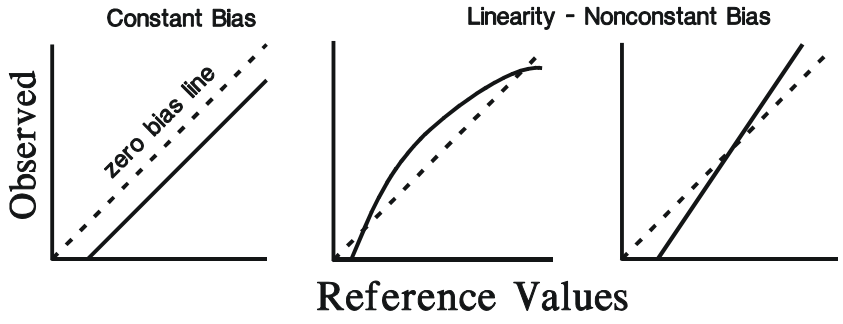
- Instrument needs calibration, reduce the calibration interval
- Worn instrument, equipment or fixture
- Normal aging or obsolescence
- Poor maintenance – air, power, hydraulic, filters, corrosion, rust, cleanliness
- Worn or damaged master, error in master
- Improper calibration or use of the setting master
- Poor quality instrument – design or conformance
- Instrument design or method lacks robustness
- Different measurement method – setup, loading, clamping, technique
- Distortion (gage or part)
- Environmental drift – temperature, humidity, vibration, cleanliness
- Violation of an assumption, error in an applied constant
- Application – part size, position, operator skill, fatigue, observation error (readability, parallax)

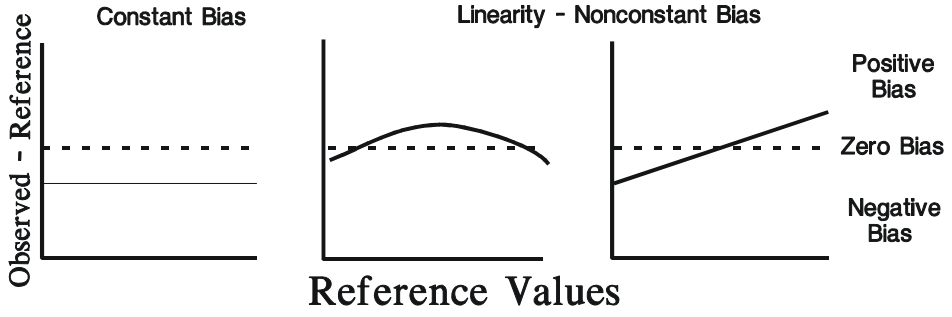
Linearity

The difference of bias throughout the **expected operating** (measurement) **range** of the equipment is called linearity. Linearity can be thought of as a change of bias with respect to size.



Note that unacceptable linearity can come in a variety of flavors. Do not assume a constant bias.





Possible causes for linearity error include:

- Instrument needs calibration, reduce the calibration interval
- Worn instrument, equipment or fixture
- Poor maintenance – air, power, hydraulic, filters, corrosion, rust, cleanliness
- Worn or damaged master(s), error in master(s) – minimum/ maximum

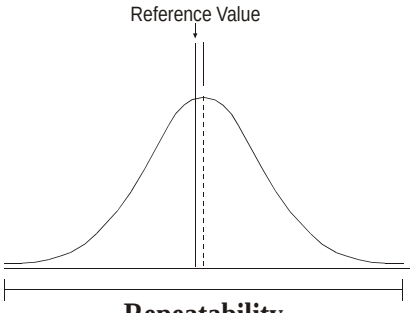
- Improper calibration (not covering the operating range) or use of the setting master(s)
- Poor quality instrument – design or conformance
- Instrument design or method lacks robustness
- Wrong gage for the application
- Different measurement method – setup, loading, clamping, technique
- Distortion (gage or part) changes with part size
- Environment – temperature, humidity, vibration, cleanliness
- Violation of an assumption, error in an applied constant
- Application – part size, position, operator skill, fatigue, observation error (readability, parallax)

Width Variation

Precision

Traditionally, precision describes the net effect of discrimination, sensitivity and repeatability over the operating range (size, range and time) of the measurement system. In some organizations precision is used interchangeably with repeatability. In fact, precision is most often used to describe the expected variation of repeated measurements over the range of measurement; that range may be size or time (i.e., "a device is as precise at the low range as high range of measurement", or "as precise today as yesterday"). One could say precision is to repeatability what linearity is to bias (although the first is random and the other systematic errors). The ASTM defines precision in a broader sense to include the variation from different readings, gages, people, labs or conditions.

Repeatability



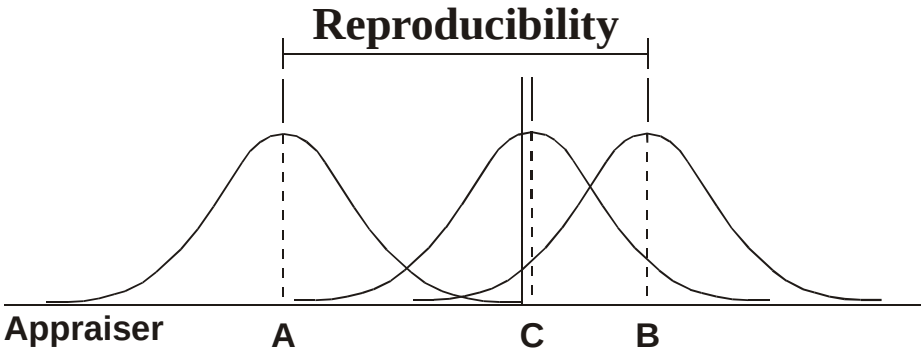
This is traditionally referred to as the "within appraiser" variability. Repeatability is the variation in measurements obtained with **one measurement instrument** when used several times by **one appraiser** while measuring the identical characteristic on the **same part.** This is the inherent variation or capability of the equipment itself. Repeatability is commonly referred to as equipment variation (*EV*), although this is misleading. In fact, repeatability is the common cause (random error) variation from successive trials under defined conditions of measurement. The best term for repeatability is *within*-system variation when the conditions of measurement are fixed and defined – fixed part, instrument, standard, method, operator, environment, and assumptions. In addition to *within*-equipment variation, repeatability will include all *within* variation (see below) from any condition in the error model.

Possible causes for poor repeatability include:

- *Within*-part (sample): form, position, surface finish, taper, sample consistency
- *Within*-instrument: repair; wear, equipment or fixture failure, poor quality or maintenance
- *Within*-standard: quality, class, wear
- *Within*-method: variation in setup, technique, zeroing, holding, clamping
- *Within*-appraiser: technique, position, lack of experience, manipulation skill or training, feel, fatigue
- *Within*-environment: short-cycle fluctuations in temperature, humidity, vibration, lighting, cleanliness
- Violation of an assumption – stable, proper operation
- Instrument design or method lacks robustness, poor uniformity
- Wrong gage for the application
- Distortion (gage or part), lack of rigidity
- Application – part size, position, observation error (readability, parallax)

Reproducibility

This is traditionally referred to as the "between appraisers" variability. Reproducibility is typically defined as the variation in the average of the measurements made by **different appraisers** using the **same measuring instrument** when measuring the identical characteristic on the **same part**. This is often true for manual instruments influenced by the skill of the operator. It is not true, however, for measurement processes (i.e., automated systems) where the operator is not a major source of variation. For this reason, reproducibility is referred to as the average variation *between*-systems or *between*-conditions of measurement.



The ASTM definition goes beyond this to potentially include not only different appraisers but also different: gages, labs and environment (temperature, humidity) as well as including repeatability in the calculation of reproducibility.

Potential sources of reproducibility error include:

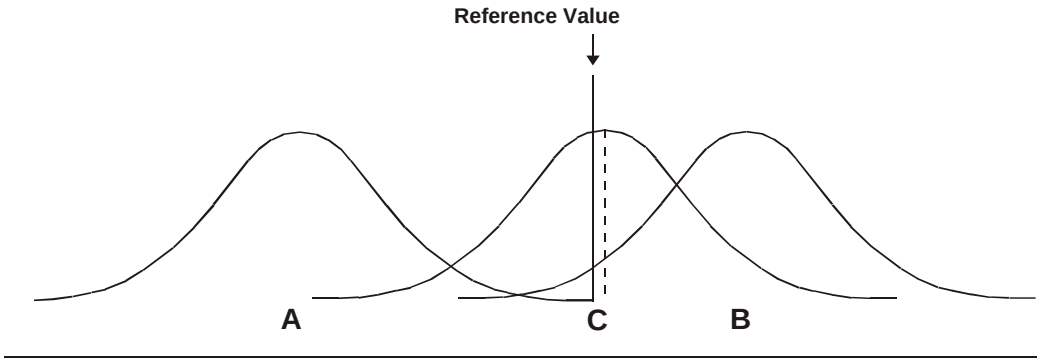
- *Between*-parts (samples): average difference when measuring types of parts A, B, C, etc, using the same instrument, operators, and method.
- *Between*-instruments: average difference using instruments A, B, C, etc., for the same parts, operators and environment. Note: in this study reproducibility error is often confounded with the method and/or operator.
- *Between*-standards: average influence of different setting standards in the measurement process.
- *Between*-methods: average difference caused by changing point densities, manual versus automated systems, zeroing, holding or clamping methods, etc.
- *Between*-appraisers (operators): average difference between appraisers A, B, C, etc., caused by training, technique, skill and experience. This is the recommended study for product and process qualification and a manual measuring instrument.
- *Between*-environment: average difference in measurements over time 1, 2, 3, etc. caused by environmental cycles; this is the most common study for highly automated systems in product and process qualifications.
- Violation of an assumption in the study
- Instrument design or method lacks robustness
- Operator training effectiveness
- Application – part size, position, observation error (readability, parallax)

> As mentioned in the two definitions above, there are differences in the definitions used by ASTM and those used by this manual. The ASTM literature focuses on interlaboratory evaluations with interest on laboratory-to-laboratory differences including the potential for different operators, gages and environment as well as within laboratory repeatability. Therefore, ASTM definitions need to encompass these differences. By ASTM standards, repeatability is the best the equipment will be under current conditions (one operator, one gage, short period of time) and reproducibility represents more typical operating conditions where there is variation from multiple sources.

Gage R&R or *GRR*

Gage R&R is an estimate of the combined variation of repeatability and reproducibility. Stated another way, *GRR* is the variance equal to the sum of within-system and between-system variances.

$$\sigma^2_{GRR} = \sigma^2_{reproducibility} + \sigma^2_{repeatability}$$



GRR

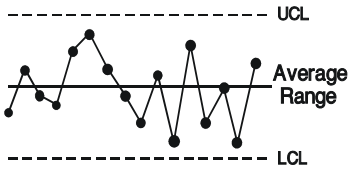
Sensitivity

Sensitivity is the smallest input that results in a detectable (usable) output signal. It is the responsiveness of the measurement system to changes in measured feature. Sensitivity is determined by gage design (discrimination), inherent quality (OEM), in-service maintenance, and the operating condition of the instrument and standard. It is always reported as a unit of measure.

Factors that affect sensitivity include:

- Ability to dampen an instrument
- Skill of operator
- Repeatability of the measuring device
- Ability to provide drift free operation in the case of electronic or pneumatic gages
- Conditions under which the instrument is being used such as ambient air, dirt, humidity

Consistency



Consistency is the difference in the variation of the measurements taken over time. It may be viewed as repeatability over time.

Factors impacting consistency are special causes of variation such as:

- Temperature of parts
- Warm up required for electronic equipment
- Worn equipment

Uniformity

Uniformity is the difference in variation throughout the operating range of the gage. It may be considered to be the homogeneity (sameness) of the repeatability over size.

Factors impacting uniformity include:

- Fixture allows smaller/larger sizes to position differently
- Poor readability on the scale
- Parallax in reading

Measurement System Variation

Capability

The capability of a measurement system is an estimate of the combined variation of measurement errors (random and systematic) based on a short-term assessment. Simple capability includes the components of:

- Uncorrected bias or linearity
- Repeatability and reproducibility (*GRR*), including short-term consistency

Refer to Chapter III for typical methods and examples to quantify each component.

An estimate of measurement capability, therefore, is an expression of the expected error for defined conditions, scope and range of the measurement system (unlike measurement *uncertainty*, which is an expression of the expected range of error or values associated with a measurement result). The capability expression of combined variation (variance) when the measurement errors are uncorrelated (random and independent) can be quantified as:

$$\sigma^2_{capability} = \sigma^2_{bias\ (linearity)} + \sigma^2_{GRR}$$

There are two essential points to understand and correctly apply measurement capability:

First, an estimate of capability is always associated with a defined scope of measurement – conditions, range and time. For example, to say that the capability of a 25 mm micrometer is 0.1 mm is incomplete without qualifying the scope and range of measurement conditions. Again, this is why an error model to define the measurement process is so important. The scope for an estimate of measurement capability could be very specific or a general statement of operation, over a limited portion or entire measurement range. Short-term could mean: the capability over a series of measurement cycles, the time to complete the *GRR* evaluation, a specified period of

production, or time represented by the calibration frequency. A statement of measurement capability need only be as complete as to reasonably replicate the conditions and range of measurement. A documented Control Plan could serve this purpose.

Second, short-term consistency and uniformity (repeatability errors) over the range of measurement are included in a capability estimate. For a simple instrument, such as a 25 mm micrometer, the repeatability over the entire range of measurement using typical, skilled operators is expected to be consistent and uniform. In this example, a capability estimate may include the entire range of measurement for multiple types of features under general conditions. Longer range or more complex measurement systems (i.e., a CMM) may demonstrate measurement errors of (uncorrected) linearity, uniformity, and short-term consistency over range or size. Because these errors are correlated they cannot be combined using the simple linear formula above. When (uncorrected) linearity, uniformity or consistency varies significantly over range, the measurement planner and analyst has only two practical choices:

1) Report the maximum (worst case) capability for the entire defined conditions, scope and range of the measurement system, or
2) Determine and report multiple capability assessments for defined portions of the measurement range (i.e., low, mid, larger range).

Performance

As with process performance, measurement system performance is the net effect of all significant and determinable sources of variation over time. Performance quantifies the long-term assessment of combined measurement errors (random and systematic). Therefore, performance includes the long-term error components of:

- Capability (short-term errors)
- Stability and consistency

Refer to Chapter III for typical methods and examples to quantify each component.

An estimate of measurement performance is an expression of the expected error for defined conditions, scope and range of the measurement system (unlike measurement *uncertainty*, which is an expression of the expected range of error or values associated with a measurement result). The performance expression of combined variation (variance) when the measurement errors are uncorrelated (random and independent) can be quantified as:

$$\sigma^2_{performance} = \sigma^2_{capability} + \sigma^2_{stability} + \sigma^2_{consistency}$$

Again, just as short-term *capability*, long-term *performance* is always associated with a defined scope of measurement – conditions, range and time. The scope for an estimate of measurement performance could be very specific or a general statement of operation, over a limited portion or entire measurement range. Long-term could mean: the average of several capability assessments over time, the long-term average error from a measurement control chart, an assessment of calibration records or multiple linearity studies, or average error from several *GRR* studies over the life and range of the measurement system. A statement of measurement performance need only be as complete as to reasonably represent the conditions and range of measurement.

Long-term consistency and uniformity (repeatability errors) over the range of measurement are included in a performance estimate. The measurement analyst must be aware of potential correlation of errors so as to not overestimate the performance estimate. This depends on how the component errors were determined. When long-term (uncorrected) linearity, uniformity or consistency vary significantly over the range, the measurement planner and analyst has only two practical choices:

1) Report the maximum (worst case) performance for the entire defined conditions, scope and range of the measurement system, or
2) Determine and report multiple performance assessments for a defined portion of the measurement range (i.e., low, mid, larger range).

Uncertainty

Measurement uncertainty is defined by VIM as a "parameter, associated with the result of a measurement, that characteristics the dispersion of the values that could reasonably be attributed to the measurand."[22] See Chapter I, Section F, for more detail.

Comments

Of a measurement system's parameters, accuracy and precision are most familiar to operating personnel since they are used in everyday life as well as technical and sales discussions. Unfortunately, these terms are also the most fuzzy as they are often thought of interchangeably. For example, if the gage is certified by an independent agency as accurate, or if the instrument is guaranteed to have high precision by the vendor, then it is incorrectly thought that all readings will fall very close to the actual values. This is not only conceptually wrong but can lead to wrong decisions about the product and process.

This ambiguity carries over to bias and repeatability (as measures of accuracy and precision). It is important to realize that:

[22] Measurand is then defined by the VIM as "the particular quantity subject to measurement".

- Bias and repeatability are independent of each other (See Figure I-E 6).
- Controlling one of these sources of error does not guarantee the control of the other. Consequently, measurement systems control programs (traditionally referred to as Gage Control Programs) ought to quantify and track all relevant sources of variation.[23]

[23] See also Chapter I, Section B.

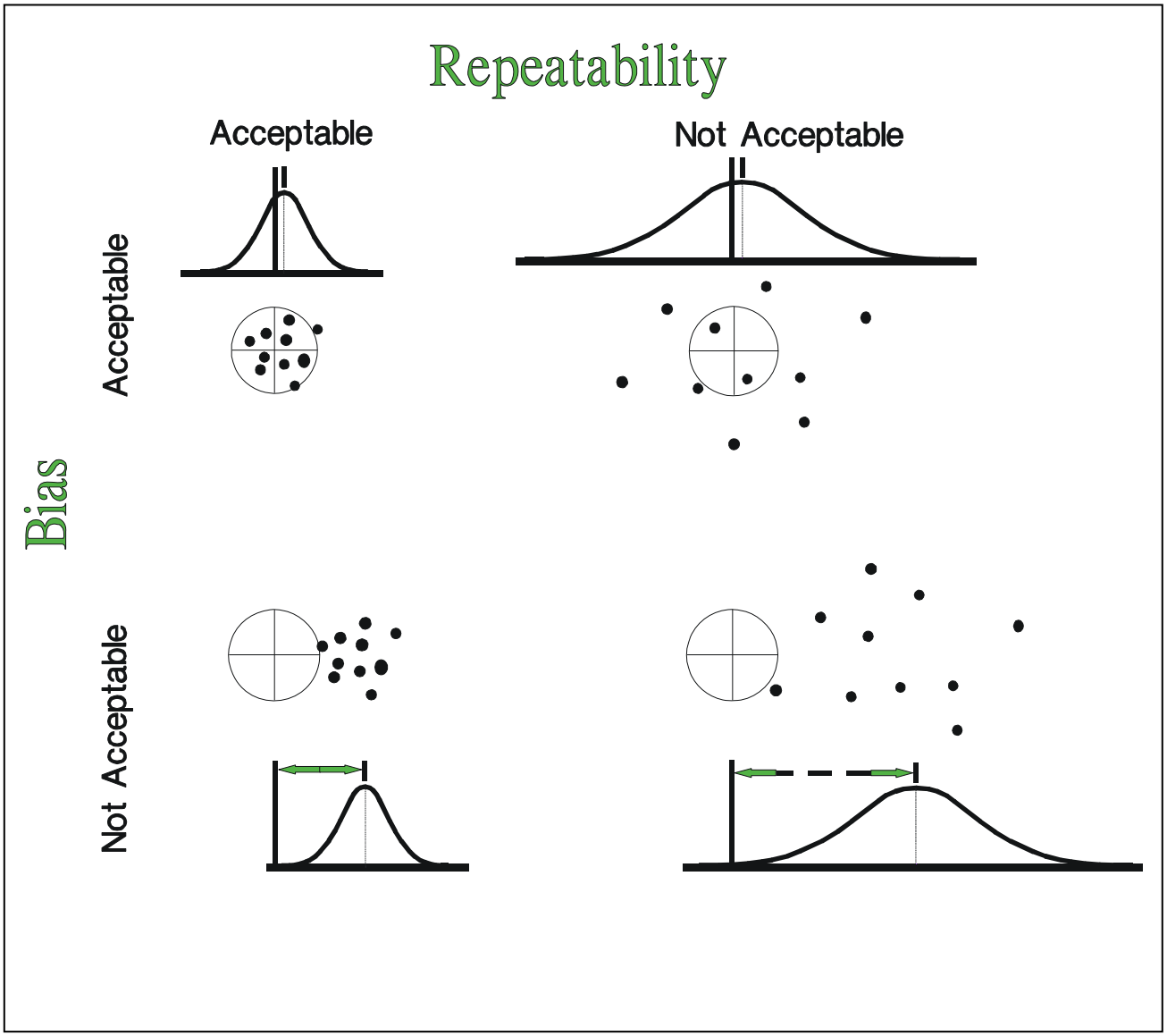


Figure I-E 6: Relationships between Bias and Repeatability

Section F
Measurement Uncertainty

Measurement Uncertainty is a term that is used internationally to describe the quality of a measurement value. While this term has traditionally been reserved for many of the high accuracy measurements performed in metrology or gage laboratories, many customer and quality system standards require that measurement uncertainty be known and consistent with required measurement capability of any inspection, measuring or test equipment.

In essence, uncertainty is the value assigned to a measurement result that describes, within a defined level of confidence, the range expected to contain the *true* measurement result. Measurement uncertainty is normally reported as a bilateral quantity. Uncertainty is a quantified expression of measurement reliability. A simple expression of this concept is:

True measurement = *observed* measurement (result) $\pm \boldsymbol{U}$

$\boldsymbol{U}$ is the term for "expanded uncertainty" of the measurand and measurement result. Expanded uncertainty is the combined standard error ($\boldsymbol{u}_c$), or standard deviation of the combined errors (random and systematic), in the measurement process multiplied by a coverage factor (k) that represents the area of the normal curve for a desired level of confidence. Remember, a normal distribution is often applied as a principle assumption for measurement systems. The ISO/IEC *Guide to the Uncertainty in Measurement* establishes the coverage factor as sufficient to report uncertainty at 95% of a normal distribution. This is often interpreted as $k = 2$.

$$\boldsymbol{U} = ku_c$$

The combined standard error ($\boldsymbol{u}_c$) includes all significant components of variation in the measurement process. In most cases, methods of measurement systems analysis performed in accordance with this manual can be used as a tool to quantify many of the sources of measurement uncertainty. Often, the most significant error component can be quantified by $\sigma^2_{performance}$. Other significant error sources may apply based on the measurement application. An uncertainty statement must include an adequate scope that identifies all significant errors and allows the measurement to be replicated. Some uncertainty statements will build from long-term, others short-term, measurement system error. However, the simple expression can be quantified as:

$$u_c^2 = \sigma^2_{performance} + \sigma^2_{other}$$

It is important to remember that measurement uncertainty is simply an estimate of how much a measurement may vary at the time of measurement. It should consider all significant sources of measurement variation in the measurement process plus significant errors of calibration, master standards, method, environment and others not previously considered in the measurement process. In many cases, this estimate will use methods of MSA and *GRR* to quantify those significant standard errors. It is appropriate to periodically reevaluate uncertainty related to a measurement process to assure the continued accuracy of the estimate.

Measurement Uncertainty and MSA

The major difference between uncertainty and the MSA is that the MSA focus is on understanding the measurement process, determining the amount of error in the process, and assessing the adequacy of the measurement system for product and process control. MSA promotes understanding and improvement (variation reduction). Uncertainty is the range of measurement values, defined by a confidence interval, associated with a measurement result and expected to include the true value of measurement.

Measurement Traceability

Traceability is the property of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties. Therefore understanding the measurement uncertainty of each link in the chain is essential. By including both the short-term and long-term sources of measurement variation that are introduced by the measurement process and the chain of traceability, the measurement system's measurement uncertainty can be evaluated assuring that all effects of traceability are taken into account. This, in turn, may reduce measurement correlation issues.

ISO Guide to the Expression of Uncertainty in Measurement

The *Guide to the Expression of Uncertainty of Measurement* (GUM) is a guide to how the uncertainty of a measurement may be evaluated and expressed. While it provides the user with an understanding of the theory and sets guidelines as to how the sources of measurement uncertainty can be classified and combined, it should be considered a high level reference document, not a "how to" manual. It does provide guidance to the user in some of the more advanced topics such as, *statistical independence* of the sources of variation, sensitivity analysis, degrees of freedom, etc. that are critical when evaluating more complex, multi-parameter measurement systems.

Section G
Measurement Problem Analysis

An understanding of measurement variation and the contribution that it makes to total variation needs to be a fundamental step in basic problem solving. When variation in the measurement system exceeds all other variables, it will become necessary to analyze and resolve those issues before working on the rest of the system. In some cases the variation contribution of the measurement system is overlooked or ignored. This may cause loss of time and resources as the focus is made on the process itself, when the reported variation is actually caused by the measurement device.

In this section a review will be made on basic problem solving steps and will show how they relate to understanding the issues in a measurement system. Each company may use the problem resolution process which the customer has approved.

If the measurement system was developed using the methods in this manual, most of the initial steps will already exist. For example, a cause and effect diagram may already exist giving valuable lessons learned about the measurement process. These data ought to be collected and evaluated prior to any formal problem solving.

Step 1

Identify the Issues

When working with measurement systems, as with any process, it is important to clearly define the problem or issue. In the case of measurement issues, it may take the form of accuracy, variation, stability, etc. The important thing to do is try to isolate the measurement variation and its contribution, from the process variation (the decision may be to work on the process, rather than work on the measurement device). The issue statement needs to be an adequate operational definition that anyone would understand and be able to act on the issue.

Step 2

Identify the Team

The problem solving team, in this case, will be dependent on the complexity of the measurement system and the issue. A simple measurement system may only require a couple of people. But as the system and issue become more complex, the team may grow in size (maximum team size ought to be limited to 10 members). The team members and the function they represent need to be identified on the problem solving sheet.

Step 3

Flowchart of Measurement System and Process

The team would review any historical flowcharting of the measurement system and the process. This would lead to discussion of known and unknown information about the measurement and its interrelationship to the process. The flowcharting process may identify additional members to add to the team.

Step 4

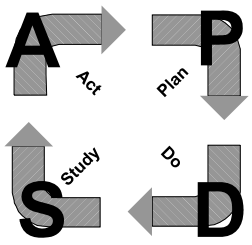
Cause and Effect Diagram

The team would review any historical Cause and Effect Diagram on the Measurement System. This could, in some cases, result in the solution or a partial solution. This would also lead to a discussion on known and unknown information. The team would use subject matter knowledge to initially identify those variables with the largest contribution to the issue. Additional studies can be done to substantiate the decisions.

Step 5

Plan-Do-Study-Act (PDSA)[24]

This would lead to a Plan-Do-Study-Act, which is form of scientific study. Experiments are planned, data are collected, stability is established, hypotheses are made and proven until an appropriate solution is reached.



Step 6

Possible Solution and Proof of the Correction

The steps and solution are documented to record the decision. A preliminary study is performed to validate the solution. This can be done using some form of design of experiment to validate the solution. Also, additional studies can be performed over time including environmental and material variation.

Step 7

Institutionalize the Change

The final solution is documented in the report; then the appropriate department and functions change the process so that the problem won't recur in the future. This may require changes in procedures, standards, and training materials. This is one of the most important steps in the process. Most issues and problems have occurred at one time or another.

[24] W. Edwards Deming, *The New Economics for Industry, Government, Education*, The MIT Press, 1994, 2000

CHAPTER II

General Concepts for Assessing Measurement Systems

Section A
Background

Two important areas need to be assessed:

1) Verify the correct variable is being measured at the proper characteristic location. Verify fixturing and clamping if applicable. Also identify any critical environmental issues that are interdependent with the measurement. If the wrong variable is being measured, then no matter how accurate or how precise the measurement system is, it will simply consume resources without providing benefit.
2) Determine what statistical properties the measurement system needs to have in order to be acceptable. In order to make that determination, it is important to know how the data are to be used, for without that knowledge, the appropriate statistical properties cannot be determined. After the statistical properties have been determined, the measurement system must be assessed to see if it actually possesses these properties or not.

Phase 1 & 2

Understand the measurement process and does it satisfy the requirements?

Phase 1 testing is an assessment to verify the correct variable is being measured at the proper characteristic location per measurement system design specification. (Verify fixturing and clamping if applicable) Also if there are any critical environmental issues that are interdependent with the measurement. Phase 1 could use a statistically designed experiment to evaluate the effect of the operating environment on the measurement system's parameters (e.g., bias, linearity, repeatability, and reproducibility). Phase 1 test results can indicate that the operating environment does not contribute significantly to the overall measurement system variation. Additionally, the variation attributable to the bias and linearity of the measurement device should be small compared with the repeatability and reproducibility components.

The knowledge gained during Phase 1 testing should be used as input to the development of the measurement system maintenance program as well as the type of testing which should be used during Phase 2. Environmental issues may drive a change in location or a controlled environment for the measurement device.

For example, if there is a significant impact of repeatability and reproducibility on the total measurement system variation, a simple two-factor statistical experiment could be performed periodically as a Phase 2 test.

Does the measurement process satisfy the requirements over time?

Phase 2 testing provides ongoing monitoring of the key sources of variation for continued confidence in the measurement system (and the data being generated) and/or a signal that the measurement system has degraded over time.

Section B
Selecting/Developing Test Procedures

"Any technique can be useful if its limitations are understood and observed."[25]

Many appropriate procedures are available for assessing measurement systems. The choice of which procedure to use depends on many factors, most of which need to be determined on a case-by-case basis for each measurement system to be assessed. In some cases, preliminary testing may be required to determine if a procedure is appropriate for a particular measurement system or not. Such preliminary testing ought to be an integral part of the Phase 1 testing discussed in the previous section.

General issues to consider when selecting or developing an assessment procedure include:

- Should standards, such as those traceable to NIST, be used in the testing and, if so, what level of standard is appropriate? Standards are frequently essential for assessing the accuracy of a measurement system. If standards are not used, the variability of the measurement system can still be assessed, but it may not be possible to assess its accuracy with reasonable credibility. Lack of such credibility may be an issue, for instance, if attempting to resolve an apparent difference between a producer's measurement system and a customer's measurement system.
- For the ongoing testing in Phase 2, the use of blind measurements may be considered. Blind measurements are measurements obtained in the actual measurement environment by an operator who does not know that an assessment of the measurement system is being conducted. Properly administered, tests based on blind measurements are usually not contaminated by the well-known Hawthorne effect.[26]
- The cost of testing.
- The time required for the testing.
- Any term for which there is no commonly accepted definition should be operationally defined. Examples of such terms include accuracy, precision, repeatability, reproducibility, etc.

---

[25] W. Edwards Deming, *The Logic of Evaluation*, The Handbook of Evaluation Research, Vol. 1, Elmer L. Struening and Marcia Guttentag, Editors

[26] The "Hawthorne Effect" refers to the outcomes of a series of industrial experiments performed at the Hawthorne Works of Western Electric between November 1924 and August 1932. In the experiments, the researchers systematically modified working conditions of five assemblers and monitored the results. As the conditions improved, production rose. However, when working conditions were degraded, production continued to improve. This was thought to be the results of the workers having developed a more positive attitude toward the work solely as a result of them being part of the study, rather than as a result of the changed working conditions. See *A History of the Hawthorne Experiments*, by Richard Gillespie, Cambridge University Press, New York, 1991.

- Will the measurements made by the measurement system be compared with measurements made by another system? If so, one should consider using test procedures that rely on the use of standards such as those discussed in Phase 1 above. If standards are not used, it may still be possible to determine whether or not the two measurement systems are working well together. However, if the systems are not working well together, then it may not be possible, without the use of standards, to determine which system needs improvement.
- How often should Phase 2 testing be performed? This decision may be based on the statistical properties of the individual measurement system and the consequence to the facility, and the facility's customers of a manufacturing process that, in effect, is not monitored due to a measurement system not performing properly.

In addition to these general issues, other issues that are specific to the particular measurement system being tested may also be important. Finding the specific issues that are important to a particular measurement system is one of the two objectives of the Phase 1 testing.

Section C
Preparation for a Measurement System Study

As in any study or analysis, sufficient planning and preparation ought to be done prior to conducting a measurement system study. Typical preparation prior to conducting the study is as follows:

1) The approach to be used should be planned. For instance, determine by using engineering judgment, visual observations, or a gage study, if there is an appraiser influence in calibrating or using the instrument. There are some measurement systems where the effect of reproducibility can be considered negligible; for example, when a button is pushed and a number is printed out.

2) The number of appraisers, number of sample parts, and number of repeat readings should be determined in advance. Some factors to be considered in this selection are:

   (a) Criticality of dimension – critical dimensions require more parts and/or trials. The reason being the degree of confidence desired for the gage study estimations.

   (b) Part configuration – bulky or heavy parts may dictate fewer samples and more trials.

   (c) Customer requirements.

3) Since the purpose is to evaluate the total measurement system, the appraisers chosen should be selected from those who normally operate the instrument.

4) Selection of the sample parts is critical for proper analysis and depends entirely upon the design of the MSA study, purpose of the measurement system, and availability of part samples that represent the production process.

For Product Control situations where the measurement result and decision criteria determine, “conformance or nonconformance to the feature specification” (i.e., 100% inspection or sampling), samples (or standards) must be selected, but need not cover the entire process range. The assessment of the measurement system is based on the feature tolerance (i.e., %*GRR* to TOLERANCE).

For Process Control situations where the measurement result and decision criteria determine, “process stability, direction and compliance with the natural process variation” (i.e., SPC, process monitoring, capability, and process improvement), the availability of samples over the entire operating range becomes very important. An independent estimate of process variation (process capability study) is recommended when assessing the adequacy of the measurement system for process control (i.e., %*GRR* to process variation).

When an independent estimate of process variation is not available, OR to determine process direction and continued suitability of the measurement system for process control, **the sample parts must be selected from the process and represent the entire production operating range.** The variation in sample parts (*PV*) selected for MSA study is used to calculate the Total Variation (*TV*) of the study. The *TV* index (i.e., %*GRR* to *TV*) is an indicator of process direction and continued suitability of the measurement system for process control. If the sample parts DO NOT represent the production process, *TV* must be ignored in the assessment. Ignoring *TV* does not affect assessments using tolerance (product control) or an independent estimate of process variation (process control).

Samples can be selected by taking one sample per day for several days. Again, this is necessary because the parts will be treated in the analysis as if they represent the range of production variation in the process. Since each part will be measured several times, each part must be numbered for identification.

5) The instrument should have a discrimination that allows at least one-tenth of the expected process variation of the characteristic to be read directly. For example, if the characteristic's variation is 0.001, the equipment should be able to "read" a change of 0.0001.

6) Assure that the measuring method (i.e., appraiser and instrument) is measuring the dimension of the characteristic and is following the defined measurement procedure.

The manner in which a study is conducted is very important. All analyses presented in this manual assume statistical independence[27] of the individual readings. To minimize the likelihood of misleading results, the following steps need to be taken:

1) The measurements should be made in a **random order**[28] to ensure that any drift or changes that could occur will be spread randomly throughout the study. The appraisers should be unaware of which numbered part is being checked in order to avoid any possible knowledge bias. However, the person conducting the study should know which numbered part is being checked and record the data accordingly, that is Appraiser A, Part 1, first trial; Appraiser B, Part 4, second trial, etc.

2) In reading the equipment, measurement values should be recorded to the practical limit of the instrument discrimination. Mechanical devices must be read and recorded to the smallest unit of scale discrimination. For electronic readouts, the measurement plan must establish a common policy for recording the right-most significant digit of display. Analog devices should

[27] There is no correlation between readings.
[28] See Chapter III, Section B, "Randomization and Statistical Independence"

be recorded to one-half the smallest graduation or limit of sensitivity and resolution. For analog devices, if the smallest scale graduation is 0.0001", then the measurement results should be recorded to 0.00005".

3) The study should be managed and observed by a person who understands the importance of conducting a reliable study.

When developing Phase 1 or Phase 2 test programs there are several factors that need to be considered:

- What effect does the appraiser have on the measurement process? If possible, the appraisers who normally use the measurement device should be included in the study.

  Each appraiser should use the procedure – including all steps – they normally use to obtain readings. The effect of any differences between methods the appraisers use will be reflected in the Reproducibility of the measurement system.

- Is appraiser calibration of the measurement equipment likely to be a significant cause of variation? If so, the appraisers should recalibrate the equipment before each group of readings.

- How many sample parts and repeated readings are required? The number of parts required will depend upon the significance of the characteristic being measured and upon the level of confidence required in the estimate of measurement system variation.

  Although the number of appraisers, trials and parts may be varied when using the recommended practices discussed in this manual, the number of appraisers, trials and parts should remain constant between Phase 1 and Phase 2 test programs or between sequential Phase 2 tests for common measurement systems. Maintaining commonality between test programs and sequential tests will improve comparisons between the various test results.

Section D
Analysis of the Results

The results should be evaluated to determine if the measurement device is acceptable for its intended application. A measurement system should be stable before any additional analysis is valid.

Assembly or Fixture Error

Acceptability Criteria – Gage Assembly and Fixture Error

An improperly designed fixture or poorly assembled gage will increase measurement error. This is normally found when the measurements indicate or display process instability or out-of-control conditions. This may be due to excessive gage variation or poor repeatability and poor *GRR* values.

In general, the first thing to do when an apparent measurement issue exists, is to review the assembly and setup instructions to make sure the gage was properly assembled, (NOTE: this will not be in the instructions) and, for example, the clamps/probes are positioned properly and have the right load. Also, for automated measurement, verify the program follows required or expected protocol.

If problems are found in any of these areas, reset or repair the gage and fixtures, then rerun the measurement evaluation.

Location Error

Acceptability Criteria – Location Error

Location error is normally defined by analyzing bias and linearity.

In general, the bias or linearity error of a measurement system is unacceptable if it is significantly different from zero or exceeds the maximum permissible error established by the gage calibration procedure. In such cases, the measurement system should be recalibrated or an offset correction applied to minimize this error.

Width Error

Acceptability Criteria – Width Error

The criteria as to whether a measurement system's variability is satisfactory are dependent upon the percentage of the manufacturing production process variability or the part tolerance that is consumed by measurement system variation. The final acceptance criteria for specific measurement systems depend on the measurement system's environment and purpose and should be agreed to by the customer. (See Chapter I Section B- "The Effects of Measurement System Variability")

When beginning to evaluate an organization's measurement systems, it can be useful to set priorities on which measurement systems to initially focus. Since the final (total) variation is based on a combination of the process and measurement variation, ($\sigma_{Total} = \sqrt{\sigma^2_{Process} + \sigma^2_{MSA}}$), when SPC is being applied for process control or collecting process data, and the control chart indicates that the process is stable and the total variation is acceptable, the measurement system can be considered

acceptable for this use and does not require separate re-evaluation[29]. If an out-of-control condition or nonconformance is found in this situation, the first thing that should be done is to evaluate the measurement system.

For measurement systems whose purpose is to analyze a process, a general guidelines for measurement system acceptability is as follows:

GRR	Decision	Comments
Under 10 percent	Generally considered to be an acceptable measurement system.	Recommended, especially useful when trying to sort or classify parts or when tightened process control is required.
10 percent to 30 percent	May be acceptable for some applications	Decision should be based upon, for example, importance of application measurement, cost of measurement device, cost of rework or repair. Should be approved by the customer.
Over 30 percent	Considered to be unacceptable	Every effort should be made to improve the measurement system. This condition may be addressed by the use of an appropriate measurement strategy; for example, using the average result of several readings of the same part characteristic in order to reduce final measurement variation.

Table II-D 1: *GRR* Criteria

Additional Width Error Metric

Another statistic of the measurement system variability is the number of distinct categories (*ndc*)[30]. This statistic indicates the number of categories into which the measurement process can be divided. This value should be greater than or equal to 5.

Caution:

The use of the GRR guidelines as threshold criteria alone is <u>NOT</u> an acceptable practice for determining the acceptability of a measurement system.

[29] If the total process is in statistical control for both the mean and variation charts and the total variation is acceptable then it can be assumed either (1) both the actual process and measurement variability are acceptable or (2) the measurement variation is not acceptable with respect to the process variation (which is extremely small) but both are in statistical control. In either case the process is producing acceptable product. In case (2) the existence of a nonconformance or out of control condition could be a false alarm (i.e., the product is acceptable but the evaluation says it is not) which would cause an evaluation of both the measurement system and process.

[30] See Chapter I, Section E, "Measurement Issues".

Applying the guidelines as the thresholds assumes that the calculated statistics are deterministic estimates of the measurement system's variability (which they are not). Specifying the guidelines as the threshold criteria can drive the wrong behavior. For example, the supplier may be creative in achieving a low *GRR* by eliminating real life sources of variation (e.g., part to gage interaction) or manipulating the measurement study (e.g., producing parts outside the expected process variation).

Comments on the Application and Gage Acceptability

When looking at *GRR* and measurement variation it is important to look at each application individually, to see what is required and how the measurement is going to be used. For example: the required precision of temperature measurement may be different for dissimilar applications. A room thermostat can regulate the temperature for human comfort and is economically priced, but may have a *GRR* upwards to 30%. It is acceptable for this application. But in a laboratory, where small variations in temperature can impact test results, a more sophisticated temperature measurement and control is required. This thermostat will be more expensive and is required to have less variability (i.e., to have a lower *GRR*).

> The final acceptance of a measurement system should not come down to a single set of indices. The long-term performance of the measurement system should also be reviewed, for example, using graphical analysis over time.

CHAPTER III

Recommended Practices for Replicable Measurement Systems

Section A
Example Test Procedures

Examples of specific test procedures are presented in this chapter. The procedures are simple to use and can be readily applied in a production environment. As discussed previously, the test procedure which should be used to understand a measurement system and to quantify its variability depends on the sources of variation which may affect the measurement system. In many situations the major sources of variation are due to the instrument (gage/equipment), person (appraiser), and method (measurement procedure). The test procedures in this chapter are sufficient for this type of measurement system analysis.

The procedures are appropriate to use when:

- ✓ Only two factors or conditions of measurement (i.e., appraisers and parts) plus measurement system repeatability are being studied
- ✓ The effect of the variability within each part is negligible
- ✓ There is no statistical interaction between appraisers and parts
- ✓ The parts do not change functionally or dimensionally during the study, i.e., are replicable

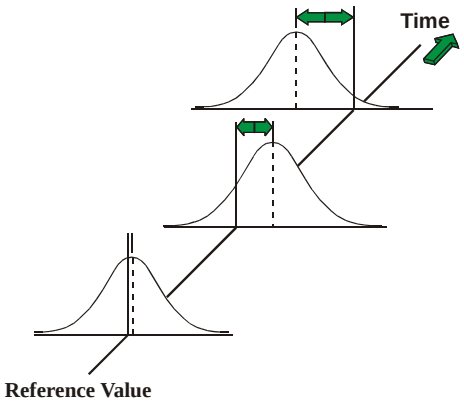
A statistical design of experiment can be conducted and/or subject matter knowledge used to determine if these procedures are appropriate for any specific measurement system.

Section B
Variable Measurement System Study Guidelines

This section contains implementation guidelines for the measurement system techniques described in Chapter I, Section E. A thorough review of Chapter I, Section E is recommended to ensure proper application of these guidelines.

Guidelines for Determining Stability

Conducting the Study



1) Obtain a sample and establish its reference value(s) relative to a traceable standard. If one is not available, select a production part[31] that falls in the mid-range of the production measurements and designate it as the master sample for stability analysis. The known reference value is not required for tracking measurement system stability.

   It may be desirable to have master samples for the low end, the high end, and the mid-range of the expected measurements. Separate measurements and control charts are recommended for each.

2) On a periodic basis (daily, weekly), measure the master sample three to five times. The sample size and frequency should be based on knowledge of the measurement system. Factors could include how often recalibration or repair has been required, how frequently the measurement system is used, and how stressful the operating conditions are. The readings need to be taken at differing times to represent when the measurement system is actually being used. This will account for warm-up, ambient or other factors that may change during the day.

3) Plot the data on an $\bar{X}$ & R or $\bar{X}$ & s control chart in time order.

Analysis of Results – Graphical

4) Establish control limits and evaluate for out-of-control or unstable conditions using standard control chart analysis.

[31] Caution should be taken where a production master could experience excessive wear due to use, material, and handling. This may require modifying the production part, such as plating, to extend the life of the master.

Analysis of Results – Numerical

Other than normal control chart analyses, there is no specific numerical analysis or index for stability.[32]

If the measurement process is stable, the data can be used to determine the bias of the measurement system.

Also, the standard deviation of the measurements can be used as an approximation for the measurement system's repeatability. This can be compared with that of the process to determine if the measurement system repeatability is suitable for the application.

> Design of Experiments or other analytical problem solving techniques may be required to determine the prime contributors to the lack of measurement system stability.

Example – Stability

To determine if the stability of a new measurement instrument was acceptable, the process team selected a part near the middle of the range of the production process. This part was sent to the measurement lab to determine the reference value which is 6.01. The team measured this part 5 times once a shift for four weeks (20 subgroups). After all the data were collected, $\overline{X}$ & *R* charts were developed (see Figure III-B 1).

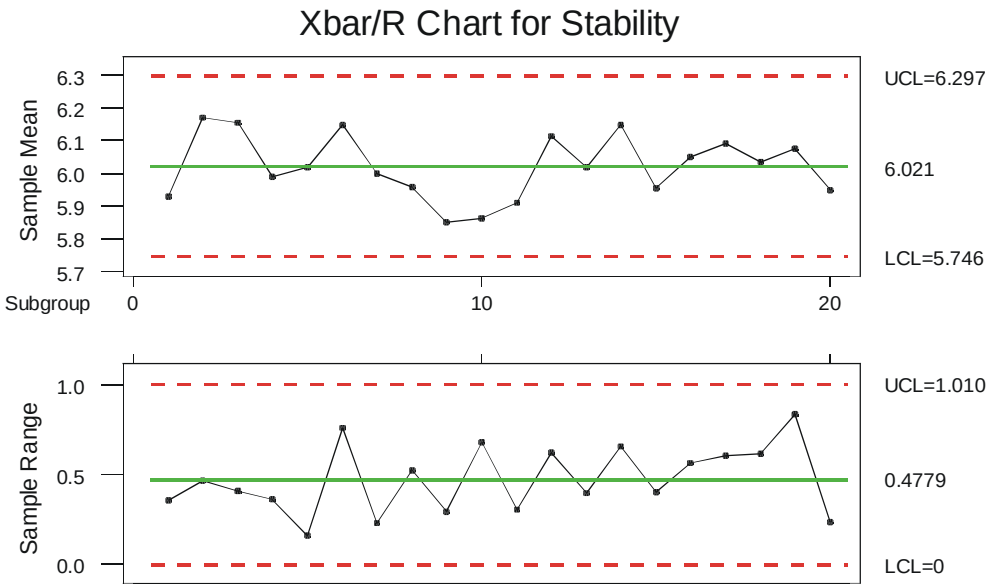


Figure III-B 1: Control Chart Analysis for Stability

Analysis of the control charts indicates that the measurement process is stable since there are no obvious special cause effects visible.

[32] See *SPC Reference Manual.*

Guidelines for Determining Bias[33] – Independent Sample Method

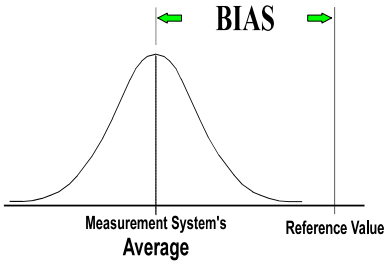
Conducting the Study

The independent sample method for determining whether the bias is acceptable uses the Test of Hypothesis:

$$H_0 \; bias = 0$$
$$H_1 \; bias \neq 0$$

The calculated average bias is evaluated to determine if the bias could be due to random (sampling) variation.

In general, the bias or linearity error of a measurement system is acceptable if it is not statistically significantly different from zero when compared to the repeatability. Consequently, the repeatability must be acceptable when compared to the process variation in order for this analysis to be useful.



1) Obtain a sample and establish its reference value relative to a traceable standard. If one is not available, select a production part that falls in the mid-range of the production measurements and designate it as the master sample for bias analysis. Measure the part $n \geq 10$ times in the gage or tool room, and compute the average of the n readings. Use this average as the "reference value."

   It may be desirable to have master samples for the low end of the expected measurements, the high end, and the mid-range. If this is done, analyze the data using a linearity study.

2) Have a single appraiser measure the sample $n \geq 10$ times in the normal manner.

Analysis of Results – Graphical

3) Determine the bias of each reading:

$$bias_i = x_i - reference \text{ value}$$

4) Plot the bias data as a histogram relative to the reference value. Review the histogram, using subject matter knowledge, to determine if any special causes or anomalies are present. If not, continue with the analysis. Special caution ought to be exercised for any interpretation or analysis when $n < 30$.

[33] See Chapter I, Section E, for an operational definition and discussion of potential causes.

Analysis of Results – Numerical

5) Compute the average bias of the *n* readings.

$$avg\ bias\ =\ \frac{\sum_{i=1}^{n} bias_i}{n}$$

6) Compute the repeatability standard deviation (see also Gage Study, Range Method, below):

$$\sigma_{repeatability} = \sigma_r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

> If a *GRR* study is available (and valid), the repeatability standard deviation calculation should be based on the study results.

7) Determine if the repeatability is acceptable by calculating the

$$\%EV\ =\ 100\ [EV/TV]\ =\ 100\ [\sigma_{repeatability}/TV]$$

Where the total variation (*TV*) is based on the expected process variation (preferred) or the specification range divided by 6 (see also *GRR* study below).

> If the %*EV* is large (see Chapter II, section D), then the measurement system variation may be unacceptable. Since the bias analysis assumes that the repeatability is acceptable, continuing the analysis with a measurement system with a large %*EV* will lead to misleading and confusing results. TF Note: What specifically are we supposed to look at in Section D that links to *EV*?

8) Determine the *t* statistic for the bias: [34]

$$\sigma_b = \sigma_r / \sqrt{n}$$

$$t\ statistic = t_{bias} = \frac{average\ bias}{\sigma_b}$$

9) Bias is acceptable (statistically zero) at the α level if

- the p-value associated with t_{bias} is less than α; or

[34] The uncertainty for bias is given by σ_b.

- zero falls within the 1-α confidence bounds based on the bias value:

$$Bias - \left[\sigma_b \left(t_{\nu, 1-\alpha/2} \right) \right] \le zero \le Bias + \left[\sigma_b \left(t_{\nu, 1-\alpha/2} \right) \right]$$

where $\nu = n - 1$ and

$t_{\nu,1-\alpha/2}$ is found using the standard t tables.

The α level which is used depends on the level of sensitivity which is needed to evaluate/control the process and is associated with the loss function (sensitivity curve) of the product/process. Customer agreement should be obtained if an α level other than the default value of .05 (95% confidence) is used.

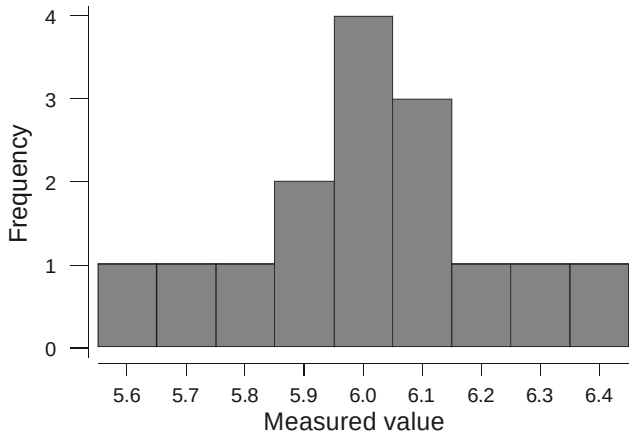
Example – Bias

A manufacturing engineer was evaluating a new measurement system for monitoring a process. An analysis of the measurement equipment indicated that there should be no linearity concerns, so the engineer had only the bias of the measurement system evaluated. A single part was chosen within the operating range of the measurement system based upon documented process variation. The part was measured by layout inspection to determine its reference value. The part was then measured fifteen times by the lead operator.

		Reference Value = 6.00	Bias
	1	5.8	-0.2
	2	5.7	-0.3
	3	5.9	-0.1
T	4	5.9	-0.1
R	5	6.0	0.0
I	6	6.1	0.1
A	7	6.0	0.0
L	8	6.1	0.1
S	9	6.4	0.4
	10	6.3	0.3
	11	6.0	0.0
	12	6.1	0.1
	13	6.2	0.2
	14	5.6	-0.4
	15	6.0	0.0

Table III-B 1: Bias Study Data

Using a spreadsheet and statistical software, the supervisor generated the histogram and numerical analysis (see Figure III-B 2 & Table III-B 2).



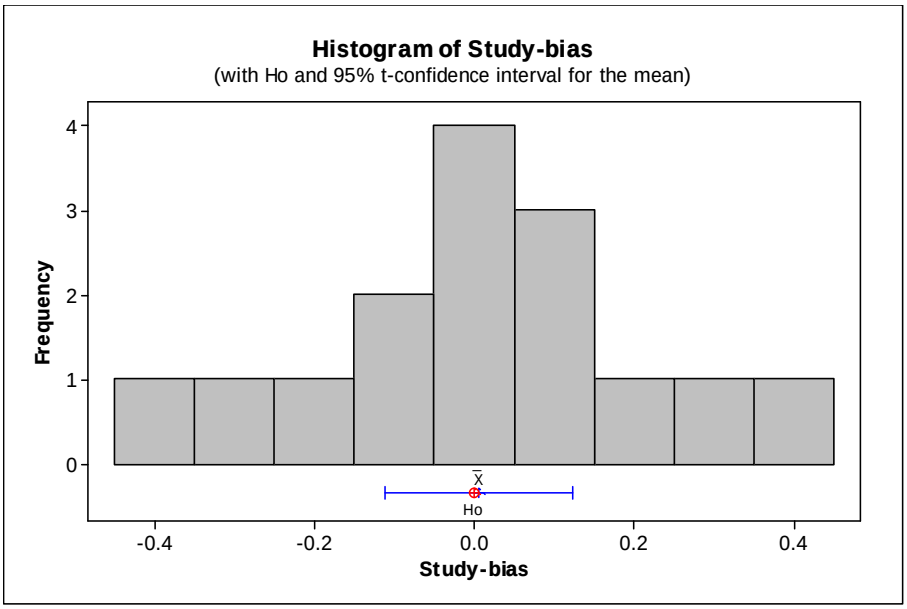


Figure III-B 2: Bias Study – Histogram of Bias Study

The histogram did not show any anomalies or outliers requiring additional analysis and review.

The repeatability of 0.2120 was compared to an expected process variation (standard deviation) of 2.5. Since the %*EV* = 100(.2120/2.5) = 8.5%, the repeatability is acceptable and the bias analysis can continue.

Since zero falls within the confidence interval of the bias (– 0.1107, 0.1241), the engineer can assume that the measurement bias is acceptable assuming that the actual use will not introduce additional sources of variation.

	n	Average	Standard Deviation, σ_r	Standard Error of Mean, σ_b
Measured Value	15	6.0067	0.2120	0.0547

	Reference Value = 6.00, α =.05					
	t statistic	*df*	Significant *t* value (2-tailed)	Average Bias	95% Confidence Interval of the Bias	
					Lower	Upper
Measured Value	0.12	14	2.14479	.0067	-0.1107	0.1241

Table III-B 2: Bias Study – Analysis of Bias Study [35]

Guidelines for Determining Bias – Control Chart Method

Conducting the Study

If an $\overline{X}$ & *R* chart is used to measure stability, the data can also be used to evaluate bias. The control chart analysis should indicate that the measurement system is stable before the bias is evaluated.

1) Obtain a sample and establish its reference value relative to a traceable standard. If one is not available, select a production part that falls in the mid-range of the production measurements and designate it as the master sample for bias analysis. Measure the part $n \geq 10$ times in the gage or tool room, and compute the average of the *n* readings. Use this average as the "reference value."
2) Conduct the stability study with *g* (subgroups) $\geq$ 20 subgroups of size *m*.

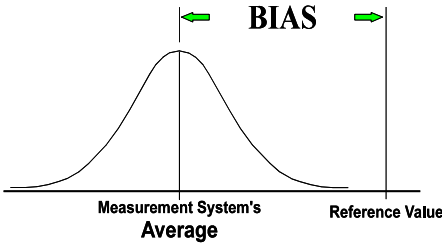
[35] Even though data is given with one digit after the decimal point, the results are shown as provided by a typical statistical program using double precision; i.e., with multiple decimal digits.

Analysis of Results – Graphical

3) If the control chart indicates that the process is stable and $m = 1$, use the analysis described for the independent sample method (see above).

4) If $m \geq 2$, plot the data as a histogram relative to the reference value. Review the histogram, using subject matter knowledge, to determine if any special causes or anomalies are present. If not, continue with the analysis.

Analysis of Results – Numerical

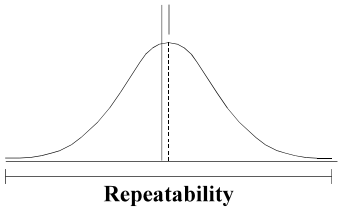


5) Obtain the $\bar{\bar{X}}$ from the control chart

6) Compute the bias by subtracting the reference value from $\bar{\bar{X}}$.

$$bias = \bar{\bar{X}} - reference\ value$$

7) Compute the repeatability standard deviation using the Average Range



$$\sigma_{repeatability} = \frac{\bar{R}}{d_2^*}$$

where d_2^* is based on the subgroup size (m) and the number of subgroups in the chart (g). (see Appendix C)

8) Determine if the repeatability is acceptable by calculating the

$$\%EV = 100\,[EV/TV] = 100\,[\sigma_{repeatability}/TV]$$

Where the total variation (TV) is based on the expected process variation (preferred) or the specification range divided by 6 (see also *GRR* study below).

If the %*EV* is large (see Chapter II, section D), then the measurement system variation may be unacceptable. Since the bias analysis assumes that the repeatability is acceptable, continuing the analysis with a measurement system with a large %*EV* will lead to misleading and confusing results; i.e., the analysis can indicate that the bias is statistically zero while the absolute magnitude of the bias exceeds acceptable equipment values.

9) Determine the *t* statistic for the bias:[36]

$$\sigma_b = \sigma_r \Big/ \sqrt{gm}$$

$$t\ statistic = t_{bias} = \frac{bias}{\sigma_b}$$

10) Bias is acceptable (statistically zero) at the α level if zero falls within the 1-α confidence bounds around the bias value:

$$Bias - \left[\sigma_b \left(t_{\nu,\, 1-\alpha/2} \right) \right] \le zero \le Bias + \left[\sigma_b \left(t_{\nu,\, 1-\alpha/2} \right) \right]$$

where ν is found in Appendix C

and $t_{\nu,1-\alpha/2}$ is found using the standard *t* tables.

> The α level which is used depends on the level of sensitivity which is needed to evaluate/control the process and is associated with the loss function (sensitivity curve) of the product/process. Customer agreement should be obtained if an α level other than the default value of .05 (95% confidence) is used.

Example – Bias

Referencing Figure III-B 1, the stability study was performed on a part which had a reference value of 6.01. The overall average of all the samples (20 subgroups of size 5 for *n*=100 samples) was 6.021. The calculated bias is therefore 0.011.

Using a spreadsheet and statistical software, the supervisor generated the numerical analysis (Table III-B 3).

Since zero falls within the confidence interval of the bias (-0.0299, 0.0519), the process team can assume that the measurement bias is acceptable assuming that the actual use will not introduce additional sources of variation.

[36] The uncertainty for bias is given by σ_b.

	n	Mean, $\bar{\bar{X}}$	Standard Deviation, σ_r	Standard Error of Mean, σ_b
Measured Value	100	6.021	.2048	.02048

	Reference Value = 6.01, α =.05, m = 5, g = 20, d_2^* = 2.334, d_2 = 2.326					
	t statistic	*df*	Significant *t* value (2-tailed)	Bias	95% Confidence Interval of the Bias	
					Lower	Upper
Measured Value	.5371	72.7	1.993	.011	-.0299	.0519

Table III-B 3: Bias Study – Analysis of Stability Study for Bias

Analysis of Bias Studies

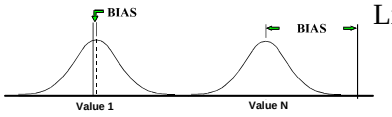
If the bias is statistically non-zero, look for these possible causes:

- Error in master or reference value. Check mastering procedure.
- Worn instrument. This can show up in the stability analysis and will suggest the maintenance or refurbishment schedule.
- Instrument made to wrong dimension.
- Instrument measuring the wrong characteristic.
- Instrument not calibrated properly. Review calibration procedure.
- Instrument used improperly by appraiser. Review measurement instructions.
- Instrument correction algorithm incorrect.

If the measurement system has non-zero bias, where possible it should be recalibrated to achieve zero bias through the modification of the hardware, software or both. If the bias cannot be adjusted to zero, it still can be used through a change in procedure (e.g., adjusting each reading by the bias). Since this has a high risk of appraiser error, it should be used only with the concurrence of the customer.

Guidelines for Determining Linearity[37]

Conducting the Study



Linearity can be evaluated using the following guidelines:

1) Select $g \geq 5$ parts whose measurements, due to process variation, cover the operating range of the gage.
2) Have each part measured by layout inspection to determine its reference value and to confirm that the operating range of the subject gage is encompassed.
3) Have each part measured $m \geq 10$ times on the subject gage by one of the operators who normally use the gage.
   - ✓ Select the parts at *random* to minimize appraiser "recall" bias in the measurements.

Analysis of Results – Graphical

4) Calculate the part bias for each measurement and the bias average for each part.

$$bias_{i,j} = x_{i,j} - \left(reference\ value\right)_i$$

$$\overline{bias_i} = \frac{\sum_{j=1}^{m} bias_{i,j}}{m}$$

5) Plot the individual biases and the bias averages with respect to the reference values on a linear graph. (see Figure III-B 3.)
6) Calculate and plot the best fit line and the confidence band of the line using the following equations.

For the best fit line, use: $\overline{y}_i = a\,x_i + b$

where

x_i = *reference value*

$\overline{y}_i$ = *bias average*

and

[37] See Chapter I, Section E, for an operational definition and discussion of potential causes.

$$a = \frac{\sum xy - \left(\frac{1}{gm}\sum x \sum y\right)}{\sum x^2 - \frac{1}{gm}\left(\sum x\right)^2} = slope$$

$$b = \bar{\bar{y}} - a\bar{x} = intercept$$

For a given x_0, the α level confidence bands[38] are:

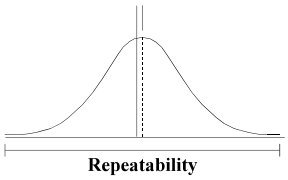
$$\text{where } s = \sqrt{\frac{\sum y_i^2 - b\sum y_i - a\sum x_i y_i}{gm-2}}$$

$$\textbf{lower: } b + a\,x_0 - \left[t_{gm-2,1-\alpha/2} \left(\frac{1}{gm} + \frac{\left(x_0 - \bar{x}\right)^2}{\sum \left(x_i - \bar{x}\right)^2} \right)^{1/2} s \right]$$

$$\textbf{upper: } b + a\,x_0 + \left[t_{gm-2,1-\alpha/2} \left(\frac{1}{gm} + \frac{\left(x_0 - \bar{x}\right)^2}{\sum \left(x_i - \bar{x}\right)^2} \right)^{1/2} s \right]$$

7) The standard deviation of the variability of repeatability.

$$\sigma_{repeatability} = s$$



Determine if the repeatability is acceptable by calculating the

$$\%EV = 100\,[EV/TV] = 100\,[\sigma_{repeatability}/TV]$$

Where the total variation (*TV*) is based on the expected process variation (preferred) or the specification range divided by 6 (see also *GRR* study below).

If the %*EV* is large (see Chapter II, section D), then the measurement system variation may be unacceptable. Since the bias analysis assumes that the repeatability is acceptable, continuing the analysis with a measurement system with a large %*EV* will lead to misleading and confusing results.

8) Plot the "*bias = 0*" line and review the graph for indications of special causes and the acceptability of the linearity. (see example Figure III-B 3.)

[38] See the note on selecting the α level in the "Guidelines for Determining Bias" area in Chapter III, Section B.

For the measurement system linearity to be acceptable, the "*bias = 0"* line must lie entirely within the confidence bands of the fitted line.

Analysis of Results – Numerical

9) If the graphical analysis indicates that the measurement system linearity is acceptable then the following hypothesis should be true:

H_0: $a = 0$ $\quad$ *slope* = 0

do not reject if

$$|t| = \frac{|a|}{\left[\frac{s}{\sqrt{\sum\left(x_j - \bar{x}\right)^2}}\right]} \leq t_{gm-2,\,1-\alpha/2}$$

If the above hypothesis is true then the measurement system has the same bias for all reference values. For the linearity to be acceptable this bias must be zero.

H_o: $b = 0$ $\quad$ *intercept (bias)* = 0

do not reject if

$$|t| = \frac{|b|}{\left[\sqrt{\frac{1}{gm} + \frac{\bar{x}^2}{\sum\left(x_i - \bar{x}\right)^2}}\right] s} \leq t_{gm-2,\,1-\alpha/2}$$

Example – Linearity

A plant supervisor was introducing a new measurement system to the process. As part of the PPAP[39] the linearity of the measurement system needed to be evaluated. Five parts were chosen throughout the operating range of the measurement system based upon documented process variation. Each part was measured by layout inspection to determine its reference value. Each part was then measured twelve times by the lead operator. The parts were selected at random during the study.

[39] *Production Parts Approval Process* manual, 4th Edition, 2006

	Part Reference Value	1 2.00	2 4.00	3 6.00	4 8.00	5 10.00
	1	2.70	5.10	5.80	7.60	9.10
	2	2.50	3.90	5.70	7.70	9.30
	3	2.40	4.20	5.90	7.80	9.50
T	4	2.50	5.00	5.90	7.70	9.30
R	5	2.70	3.80	6.00	7.80	9.40
I	6	2.30	3.90	6.10	7.80	9.50
A	7	2.50	3.90	6.00	7.80	9.50
L	8	2.50	3.90	6.10	7.70	9.50
S	9	2.40	3.90	6.40	7.80	9.60
	10	2.40	4.00	6.30	7.50	9.20
	11	2.60	4.10	6.00	7.60	9.30
	12	2.40	3.80	6.10	7.70	9.40

Table III-B 4: Linearity Study Data

Using a spreadsheet and statistical software, the supervisor generated the linearity plot (Figure III-B 3).

	Part Reference Value	1 2.00	2 4.00	3 6.00	4 8.00	5 10.00
	1	0.7	1.1	-0.2	-0.4	-0.9
	2	0.5	-0.1	-0.3	-0.3	-0.7
	3	0.4	0.2	-0.1	-0.2	-0.5
B	4	0.5	1.0	-0.1	-0.3	-0.7
I	5	0.7	-0.2	0.0	-0.2	-0.6
A	6	0.3	-0.1	0.1	-0.2	-0.5
S	7	0.5	-0.1	0.0	-0.2	-0.5
	8	0.5	-0.1	0.1	-0.3	-0.5
	9	0.4	-0.1	0.4	-0.2	-0.4
	10	0.4	0.0	0.3	-0.5	-0.8
	11	0.6	0.1	0.0	-0.4	-0.7
	12	0.4	-0.2	0.1	-0.3	-0.6
	BIAS Avg.	0.491667	0.125	0.025	-0.29167	-0.61667

Table III-B 5: Linearity Study – Intermediate Results

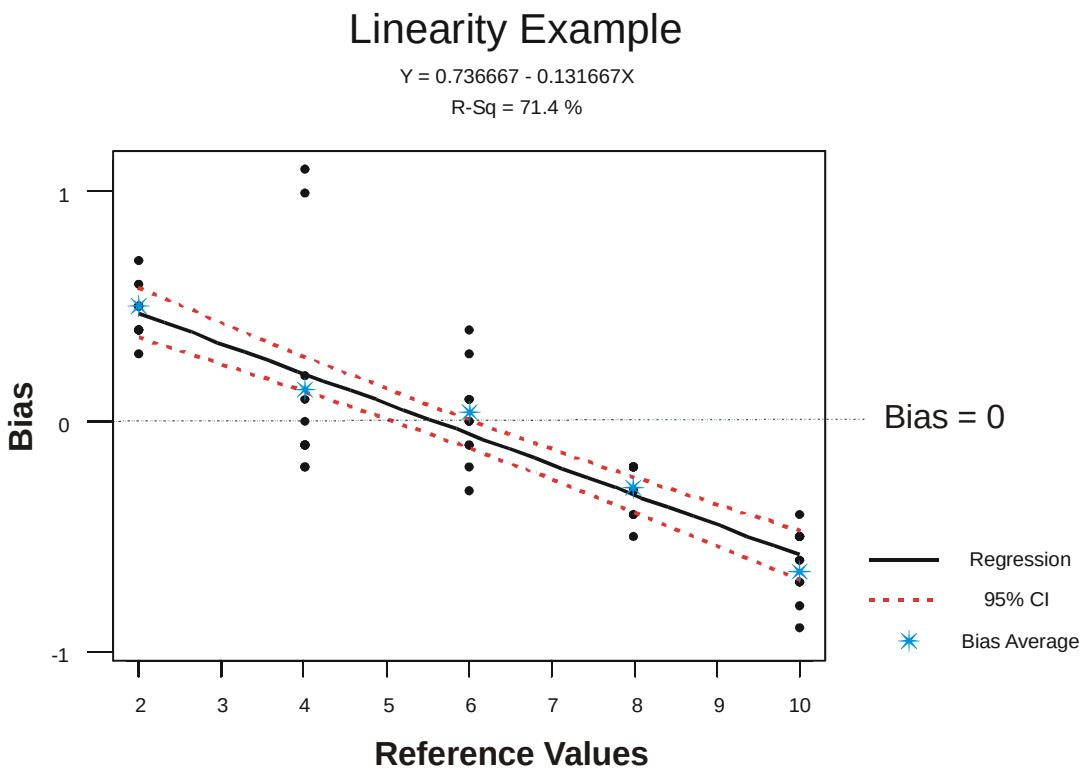


Figure III-B 3: Linearity Study – Graphical Analysis

The graphical analysis indicates that special causes may be influencing the measurements system. The data for reference value 4 appear to be bimodal.

Even if the data for reference value 4 were not considered, the graphical analysis clearly shows that this measurement system has a linearity problem. The R^2 value indicates that a linear model may not be an appropriate model for these data.[40] Even if the linear model is accepted, the "*bias = 0*" line intersects the confidence bounds rather than being contained by them.

At this point, the supervisor ought to begin problem analysis and resolution on the measurement system, since the numerical analysis will not provide any additional insights. However, wanting to make sure no paperwork is left unmarked, the supervisor calculates the *t*-statistic for the slope and intercept:

$$t_a = -12.043$$

$$t_b = 10.158$$

Taking the default $\alpha = .05$ and going to the *t*-tables with $(gm - 2) = 58$ degrees of freedom and a proportion of .975, the supervisor comes up with the critical value of:

[40] See standard statistical texts on analysis of the appropriateness of using a linear model to describe the relationship between two variables.

$$t_{58,\ .975} = 2.00172$$

Since $|t_a| > t_{58,\ .975}$, the result obtained from the graphical analysis is reinforced by the numerical analysis – there is a linearity problem with this measurement system.

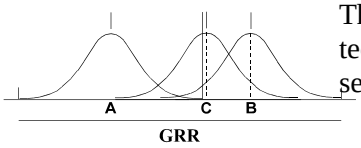
In this case, it does not matter what relation t_b has to $t_{58,\ .975}$ since there is a linearity problem. Possible causes for linearity problems can be found in Chapter I, Section E, "Location Variation."

If the measurement system has a linearity problem, it needs to be recalibrated to achieve zero bias through the modification of the hardware, software or both.

If the bias cannot be adjusted to zero bias throughout the measurement system range, it still can be used for product/process control but not analysis as long as the measurement system remains stable.

> Since this has a high risk of appraiser error, it should be used only with the concurrence of the customer.

Guidelines for Determining Repeatability and Reproducibility[41]



The Variable Gage Study can be performed using a number of differing techniques. Three acceptable methods will be discussed in detail in this section. These are:

- Range method
- Average and Range method (including the Control Chart method)
- ANOVA method

Except for the Range method, the study data design is very similar for each of these methods. The ANOVA method is preferred because it measures the operator to part interaction gauge error, whereas the Range and the Average and Range methods does not include this variation. As presented, all methods ignore within-part variation (such as roundness, diametric taper, flatness, etc., as discussed in Chapter IV, Section D) in their analyses.

> The ANOVA approach can identify appraiser-part interaction but it can also evaluate other sources of variation which is the reason why it was included. Historically, the assumption is made that the interaction is zero, in which case the results of both approaches are equivalent. With that said, the ANOVA approach is preferred because of its flexibility if the user has access to a appropriate computer program. If not, the X bar and R approach is appropriate and can be done manually or via a computer program.

[41] See Chapter I, Section E, for an operational definition and discussion of potential causes.

However, the total measurement system includes not only the gage itself and its related bias, repeatability, etc., but also could include the variation of the parts being checked. The determination of how to handle within-part variation needs to be based on a rational understanding of the intended use of the part and the purpose of the measurement.

Finally, all of the techniques in this section are subject to the prerequisite of statistical stability.

> Although reproducibility is usually interpreted as appraiser variation, there are situations when this variation is due to other sources of variation. For example, with some in-process measurement systems there are no human appraisers. If all the parts are handled, fixtured and measured by the same equipment, then reproducibility is zero; i.e., only a repeatability study is needed. If, however, multiple fixtures are used, then the reproducibility is the between-fixture variation.

Range Method

The Range method is a modified variable gage study which will provide a quick **approximation** of measurement variability. This method will provide **only** the overall picture of the measurement system. It **does not** decompose the variability into repeatability and reproducibility. It is typically used as a quick check to verify that the *GRR* has not changed.

> This approach has the potential to detect an unacceptable measurement system[42] 80% of the time with a sample size of 5 and 90% of the time with a sample size of 10.

The Range method typically uses two appraisers and five parts for the study. In this study, both appraisers measure each part once. The range for each part is the absolute difference between the measurement obtained by appraiser A and the measurement obtained by appraiser B. The sum of the ranges is found and the average range ($\overline{R}$) is calculated. The total measurement variability is found by multiplying the average range by $\frac{1}{d_2^*}$ where d_2^* is found in Appendix C, with m = 2 and g = number of parts.

Parts	Appraiser A	Appraiser B	Range (A, B)
1	0.85	0.80	0.05
2	0.75	0.70	0.05
3	1.00	0.95	0.05
4	0.45	0.55	0.10
5	0.50	0.60	0.10

[42] i.e., *%GRR* > 30%

$$AverageRange\ (\bar{R}) = \frac{\sum R_i}{5} = \frac{0.35}{5} = 0.07$$

$$GRR = \left(\frac{\bar{R}}{d_2^*}\right) = \left(\frac{\bar{R}}{1.19}\right) = \left(\frac{0.07}{1.19}\right) = 0.0588$$

(Process Standard Deviation = 0.0777 from previous study)

$$\%GRR = 100 * \left(\frac{GRR}{Process\ Standard\ Deviation}\right) = 75.7\%$$

Table III-B 6: Gage Study (Range Method)

To determine what percentage of the process standard deviation the measurement variation consumes, convert the *GRR* to a percentage by multiplying by 100 and dividing by the process standard deviation. In the example (see Table III-B 6), the process standard deviation for this characteristic is 0.0777, therefore,

$$\%GRR = 100 * \left(\frac{GRR}{Process\ Standard\ Deviation}\right) = 75.7\%$$

Now that the *%GRR* for the measurement system is determined, an interpretation of the results should be made. In Table III-B 6, the *%GRR* is determined to be 75.7% and the conclusion is that the measurement system is in need of improvement.

Average and Range Method

The Average and Range method ($\bar{X}$ & *R*) is an approach which will provide an estimate of both repeatability and reproducibility for a measurement system. Unlike the Range method, this approach will allow the measurement system's variation to be decomposed into two separate components, repeatability and reproducibility.[43] However, variation due to the interaction between the appraiser and the part/gage is not accounted for in the analysis.

[43] The *ANOVA* method can be used to determine the interaction between the gage and appraisers, if such exists.

Conducting the Study

Although the number of appraisers, trials and parts may be varied, the subsequent discussion represents the optimum conditions for conducting the study. Refer to the *GRR* data sheet in III-B 6a. The detailed procedure is as follows:

1) Obtain a sample of $n \geq 10$ parts[44] that represent the actual or expected range of process variation.

2) Refer to the appraisers as A, B, C, etc. and number the parts 1 through *n* so that the numbers are not visible to the appraisers.

See Chapter II, Section C.

3) Calibrate the gage if this is part of the normal measurement system procedures. Let appraiser A measure *n* parts in a *random* order[45] and enter the results in row 1.

4) Let appraisers B and C measure the same *n* parts without seeing each other's readings; then enter the results in rows 6 and 11, respectively.

5) Repeat the cycle using a different random order of measurement. Enter data in rows 2, 7 and 12. Record the data in the appropriate column. For example if the first piece measured is part 7 then record the result in the column labeled part 7. If three trials are needed, repeat the cycle and enter data in rows 3, 8 and 13.

6) Steps 4 and 5 may be changed to the following when large part size or simultaneous unavailability of parts makes it necessary:

   - ✓ Let appraiser A measure the first part and record the reading in row 1. Let appraiser B measure the first part and record the reading in row 6. Let appraiser C measure the first part and record the reading in row 11.
   - ✓ Let appraiser A repeat reading on the first part and record the reading in row 2, appraiser B record the repeat reading in row 7, and appraiser C record the repeat reading in row 12. Repeat this cycle and enter the results in rows 3, 8, and 13, if three trials are to be used.

7) An alternative method may be used if the appraisers are on different shifts. Let appraiser A measure all 10 parts and enter the reading in row 1. Then have appraiser A repeat the reading in a different order and enter the results in rows 2 and 3. Do the same with appraisers B and C.

---

[44] The total number of "ranges" generated ought to be > 15 for a minimal level of confidence in the results. Although the form was designed with a maximum of 10 parts, this approach is not limited by that number. As with any statistical technique, the larger the sample size, the less sampling variation and less resultant risk will be present.

[45] See Chapter III, Section B, "Randomization and Statistical Independence"

Gage Repeatability and Reproducibility Data Collection Sheet

	Appraiser /Trial #		PART 1	2	3	4	5	6	7	8	9	10	AVERAGE
1	A	1	0.29	-0.56	1.34	0.47	-0.80	0.02	0.59	-0.31	2.26	-1.36	
2		2	0.41	-0.68	1.17	0.50	-0.92	-0.11	0.75	-0.20	1.99	-1.25	
3		3	0.64	-0.58	1.27	0.64	-0.84	-0.21	0.66	-0.17	2.01	-1.31	
4		Average											$\bar{X}_a$ =
5		Range											$\bar{R}_a$ =
6	B	1	0.08	-0.47	1.19	0.01	-0.56	-0.20	0.47	-0.63	1.80	-1.68	
7		2	0.25	-1.22	0.94	1.03	-1.20	0.22	0.55	0.08	2.12	-1.62	
8		3	0.07	-0.68	1.34	0.20	-1.28	0.06	0.83	-0.34	2.19	-1.50	
9		Average											$\bar{X}_b$ =
10		Range											$\bar{R}_b$ =
11	C	1	0.04	-1.38	0.88	0.14	-1.46	-0.29	0.02	-0.46	1.77	-1.49	
12		2	-0.11	-1.13	1.09	0.20	-1.07	-0.67	0.01	-0.56	1.45	-1.77	
13		3	-0.15	-0.96	0.67	0.11	-1.45	-0.49	0.21	-0.49	1.87	-2.16	
14		Average											$\bar{X}_c$ =
15		Range											$\bar{R}_c$ =
16	Part Average												$\bar{\bar{X}}$ = R_p =
17	$\bar{\bar{R}} = ([\bar{R}_a =$]+[$\bar{R}_b =$]+[$\bar{R}_c =$])/[# *OF APPRAISERS* =] =												$\bar{\bar{R}}$ =
18	$\bar{X}_{DIFF}$ = [*Max* $\bar{X}$ =] - [*Min* $\bar{X}$ =] =												
19	* $UCL_R = [\bar{\bar{R}} =$] × [$D_4 =$] =												

*D_4 = 3.27 for 2 trials and 2.58 for 3 trials. UCL_R represents the limit of individual *R*'s. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and re-average and recompute $\bar{\bar{R}}$ and the limiting value from the remaining observations.

Notes:_____

Table III-B 6a: Gage Repeatability and Reproducibility Data Collection Sheet

Analysis of Results – Graphical[46]

The use of graphical tools is very important. The specific graphical tools used depend on the experimental design employed to collect the data. A systematic screening of the data for apparent special causes of variations by using graphical tools should precede any other statistical analysis.

The following are some of the techniques which have proven to be useful. (See also the Analysis of Variance Method).

The data from the measurement system analysis can be displayed graphically by control charts. The idea of using control charts to answer questions concerning the measurement system has been used by Western Electric (see Reference List: *AT&T Statistical Quality Control Handbook*).

Average Chart

The averages of the multiple readings by each appraiser on each part are plotted by appraiser with part number as an index. This can assist in determining consistency between appraisers.

The grand average and control limits determined by using the average range are also plotted. The resulting Average Chart provides an indication of "usability" of the measurement system.

The area within the control limits represents the measurement sensitivity ("noise"). Since the group of parts used in the study represents the process variation, approximately one half or more of the averages should fall outside the control limits. If the data show this pattern, then the measurement system should be adequate to detect part-to-part variation and the measurement system can provide useful information for analyzing and controlling the process. If less than half fall outside the control limits then either the measurement system lacks adequate effective resolution or the sample does not represent the expected process variation.

[46] Detailed descriptions of these analyses are beyond the scope of this document. For more information, refer to the references and seek assistance from competent statistical resources

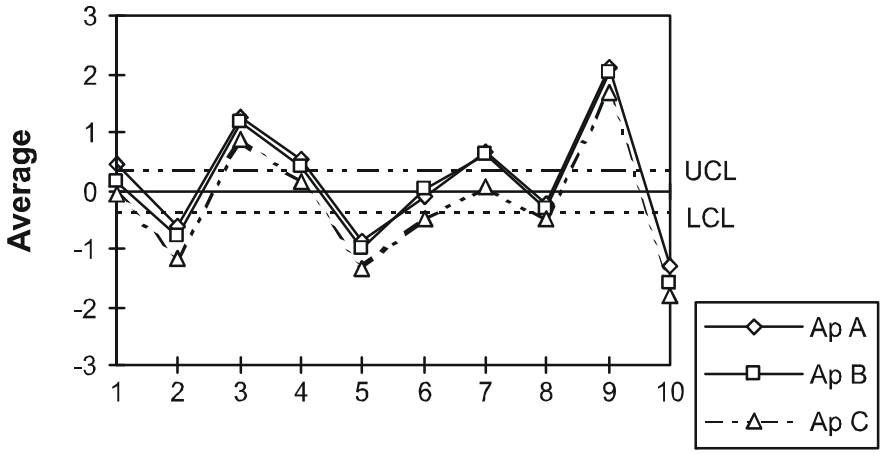


Figure III-B 4: Average Chart – "Stacked"[47]

Review of the charts indicates that the measurement system appears to have sufficient discrimination for processes with variation described by the sample parts. No appraiser-to-appraiser differences are readily apparent.

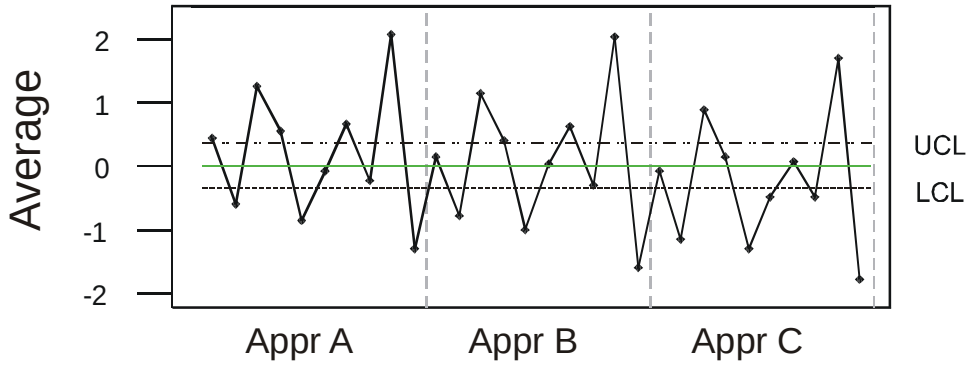


Figure III-B 5: Average Chart – "Unstacked"

[47] With the *ANOVA* approach, this is also referred to as *appraiser-by-part interaction* chart

Range Chart

The range control chart is used to determine whether the process is in control. The reason being that no matter how large the measurement error may be, the control limits will allow for that error. That is why the special causes need to be identified and removed before a measurement study can be relevant.

The ranges of the multiple readings by each appraiser on each part are plotted on a standard range chart including the average range and control limit(s). From the analysis of the data that are being plotted, several useful interpretations can be made. If all ranges are in control, all appraisers are doing the same job.

If one appraiser is out-of-control, the method used differs from the others.

If all appraisers have some out of control ranges, the measurement system is sensitive to appraiser technique and needs improvement to obtain useful data.

> Neither chart should display patterns in the data relative to the appraisers or parts.
>
> The ranges are not ordered data. Normal control chart trend analysis must not be used even if the plotted points are connected by lines.
>
> Stability is determined by a point or points beyond the control limit; within-appraiser or within-part patterns. Analysis for stability ought to consider practical and statistical significance.

The range chart can assist in determining:

- Statistical control with respect to repeatability
- Consistency of the measurement process between appraisers for each part

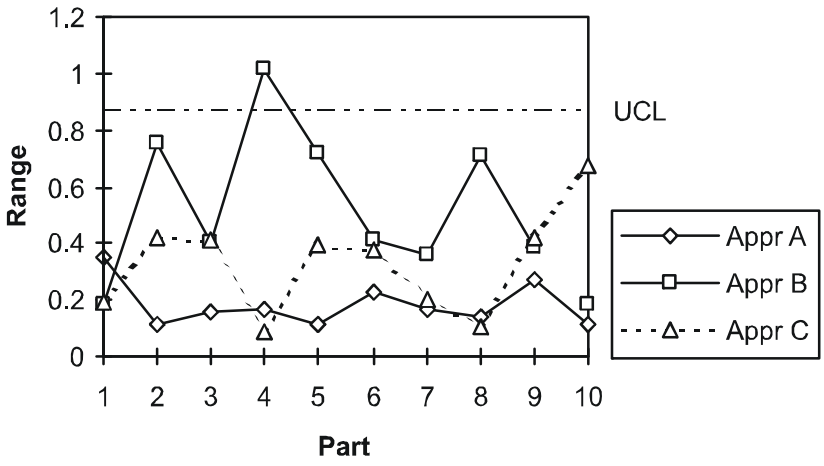


Figure III-B 6: Range Chart – "Stacked"

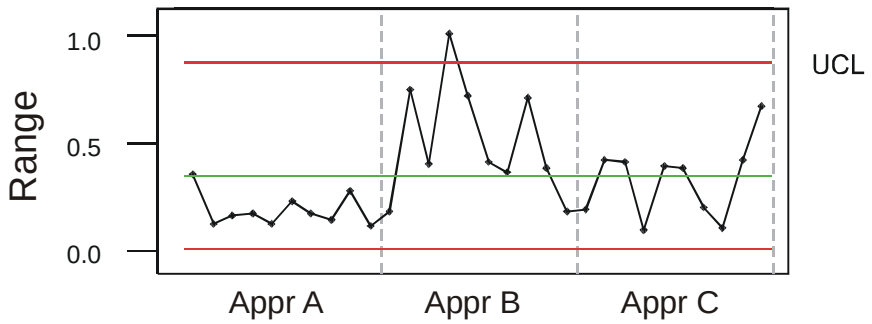


Figure III-B 7: Range Chart – "Unstacked"

Review of the above charts indicates that there are differences between the variability of the appraisers.

Run Chart

The individual readings are plotted by part for all appraisers (see Figure III-B 8) to gain insight into:

- The effect of individual parts on variation consistency
- Indication of outlier readings (i.e., abnormal readings)

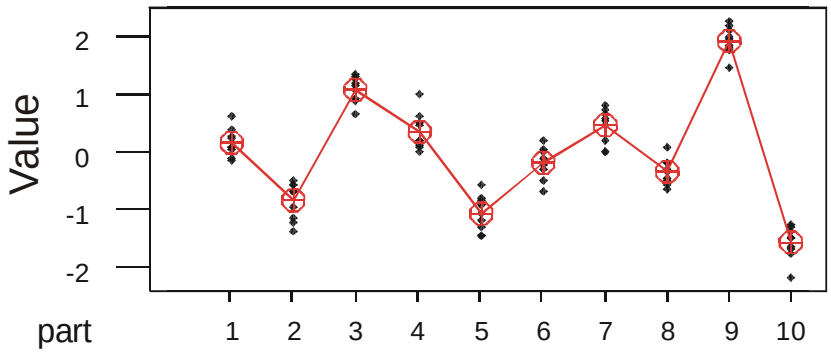


Figure III-B 8: Run Chart by Part

Review of the above chart does not indicate any outliers or inconsistent parts.

Scatter Plot

The individual readings are plotted by part-by-appraiser (see Figure III-B 9) to gain insight into:

- Consistency between appraisers
- Indication of possible outliers
- Part-appraiser interactions

Review of the Figure III-B 9 does not indicate any significant outliers but does indicate that appraiser C may have lower readings than the other appraisers.

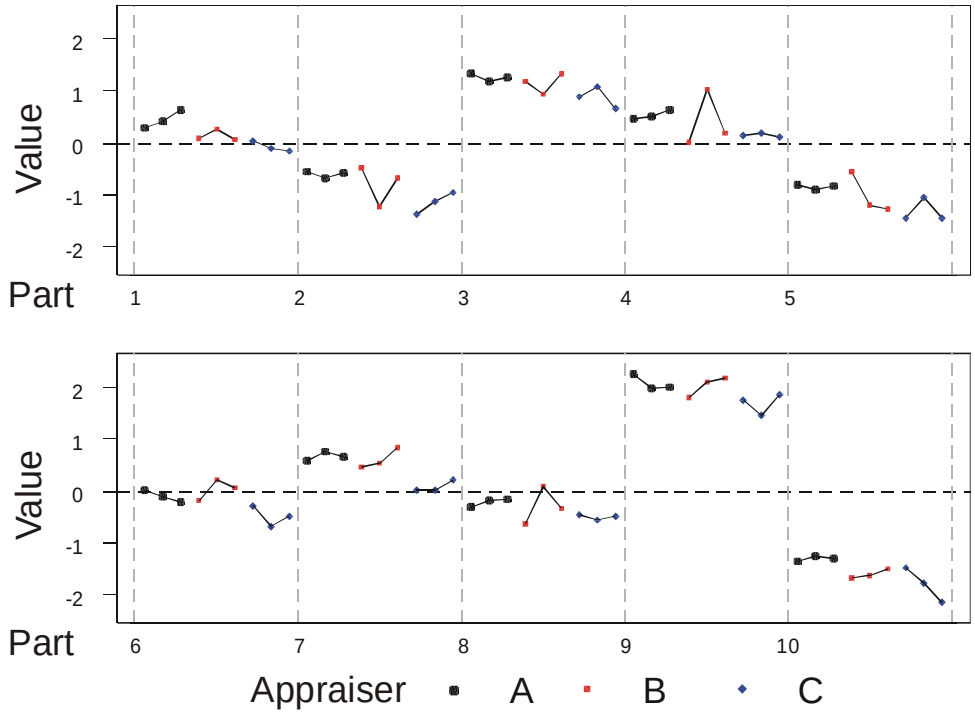


Figure III-B 9: Scatter Plot

Whiskers Chart

In a Whiskers Chart, the high and low data values and the average by part-by-appraiser are plotted (see Figure III-B 10). This provides insight into:

- Consistency between appraisers
- Indication of outliers
- Part-appraiser interactions

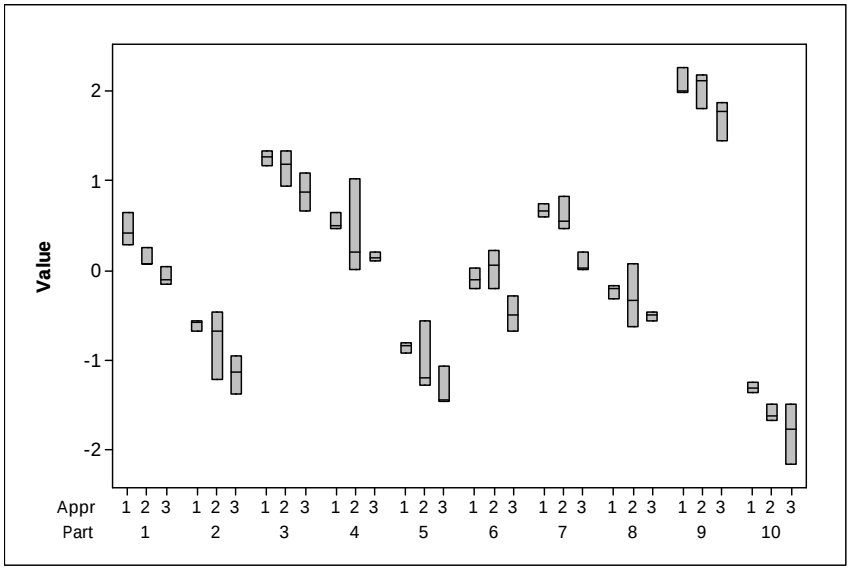


Figure III-B 10: Whiskers Chart

Review of Figure III-B 10 does not indicate any significant outliers but indicates that appraiser B may have the most variability.

Error chart

The data from the measurement system analysis can be analyzed by running "Error Charts" (see Figure III-B 11) of the individual deviations from the accepted reference values. The individual deviation or error for each part is calculated as follows:

Error = Observed Value – Reference Value

or

Error = Observed Value – Average Measurement of the Part

This depends upon whether or not reference values of the data being measured are available.

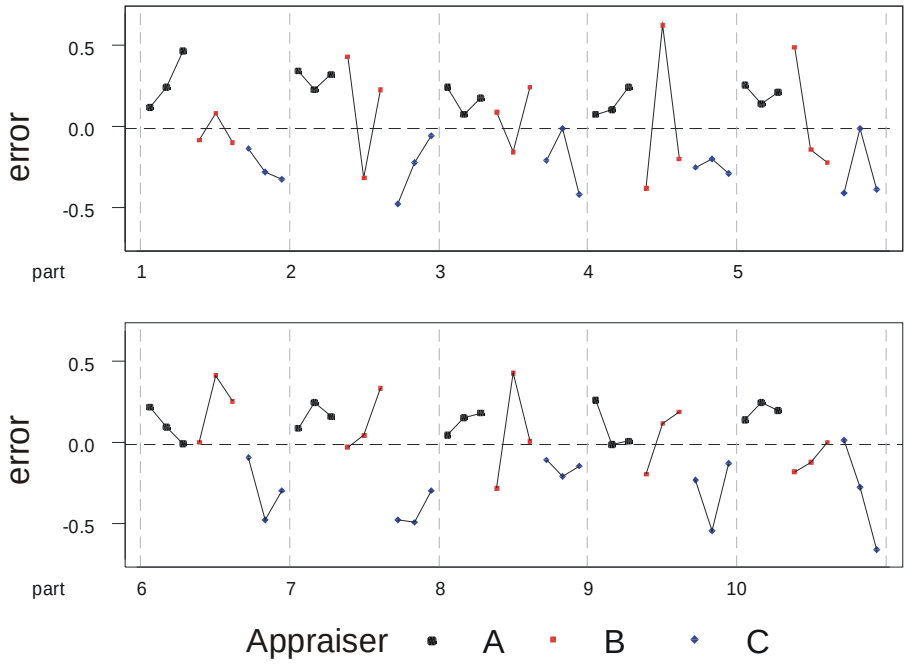


Figure III-B 11: Error Charts

Review of the above charts indicates:

- Appraiser A has an overall positive bias
- Appraiser B has the most variability but no apparent bias.
- Appraiser C has an overall negative bias

Normalized Histogram

The histogram plot (Figure III-B 12) is a graph that displays the frequency distribution of the gage error of appraisers who participated in the study. It also shows their combined frequency distribution.

If the reference values are available:

Error = Observed Value – Reference Value

Otherwise:

Normalized Value = Observed Value – Part Average

The histogram plot provides a quick visual overview of how the error is distributed. Issues such as whether bias or lack of consistency exists in the measurements taken by the appraisers, can be identified even before the data are analyzed.

Analysis of the histograms (Figure III-B 12) reinforces that of the error charts. They also indicate that only appraiser B has a symmetric form. This may indicate that appraisers A and C are introducing a systematic source of variation which is resulting in the biases.

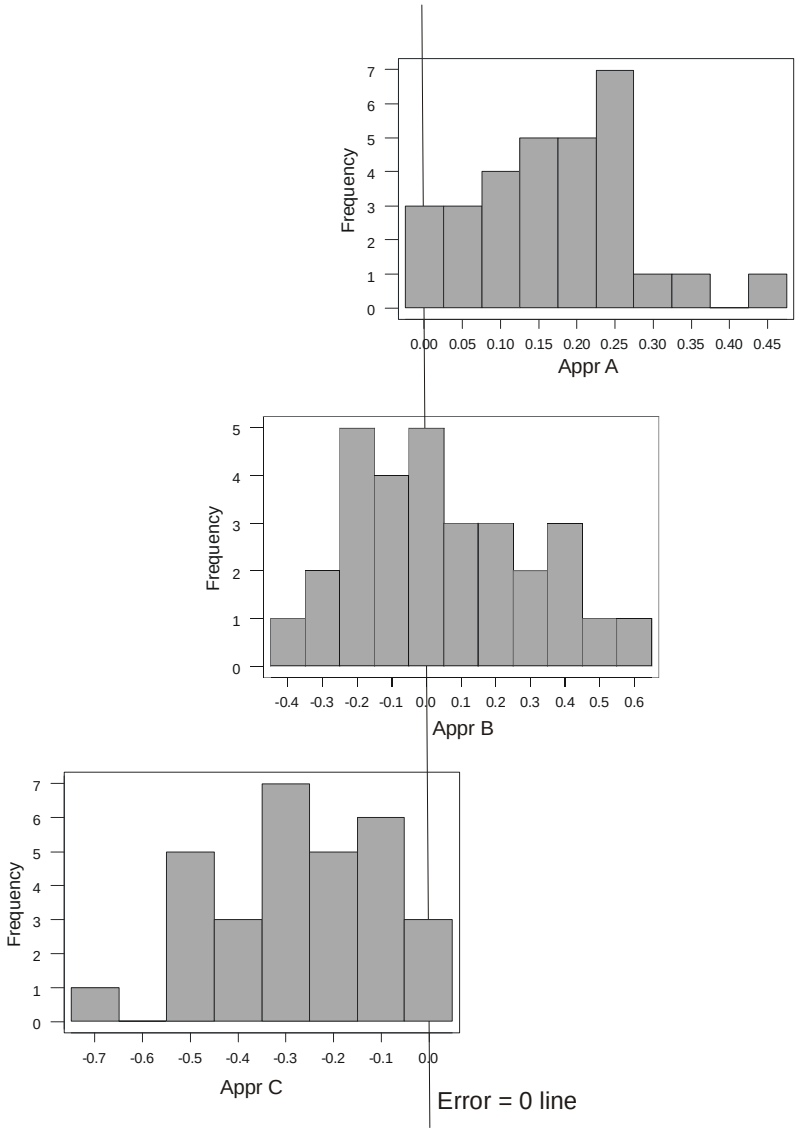


Figure III-B 12: Normalized Histogram[48]

[48] Note that the "0.0" of each of the histograms are aligned with respect to each other.

X-Y Plot of Averages by Size

The averages of the multiple readings by each appraiser on each part are plotted with the reference value or overall part averages as the index (see Figure III-B 13). This plot can assist in determining:

- Linearity (if the reference value is used)
- Consistency in linearity between appraisers

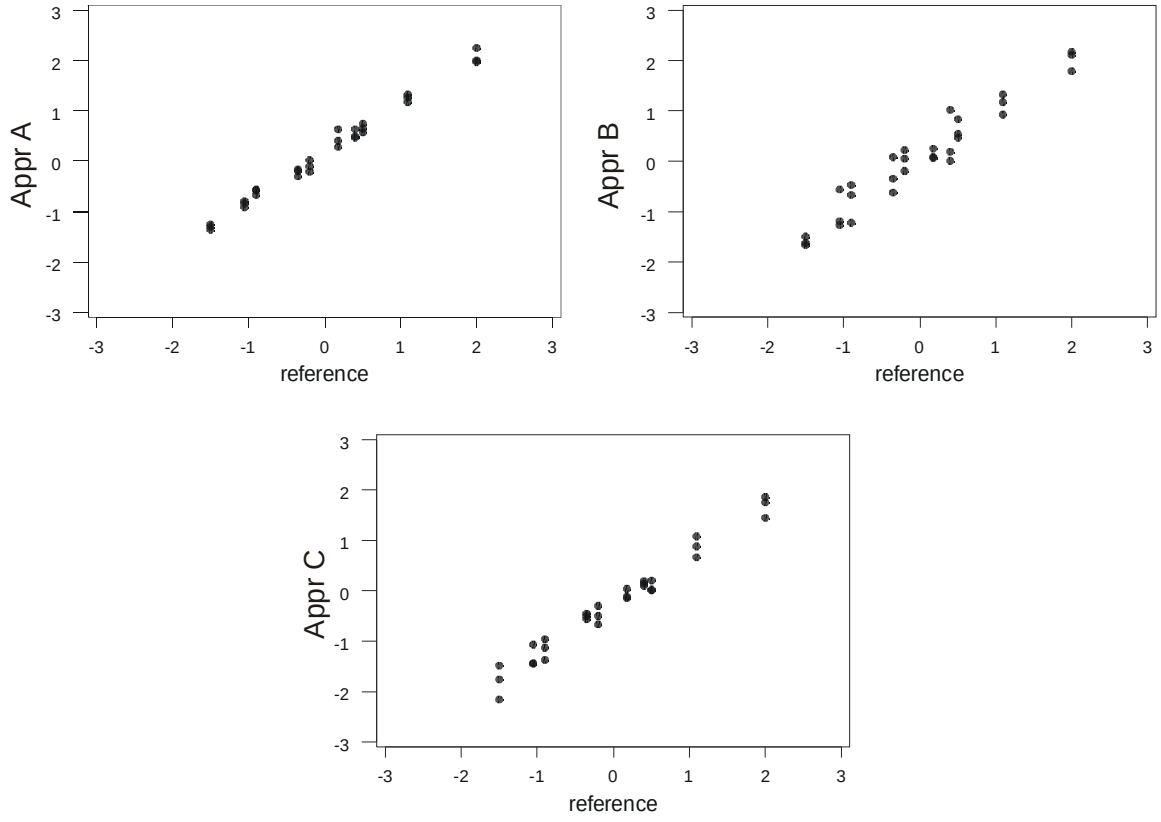


Figure III-B 13: X–Y Plot of Averages by Size

Comparison X-Y Plots

The averages of the multiple readings by each appraiser on each part are plotted against each other with the appraisers as indices. This plot compares the values obtained by one appraiser to those of another (see Figure III-B 14). If there were perfect agreement between appraisers, the plotted points would describe a straight line through the origin and 45° to the axis.

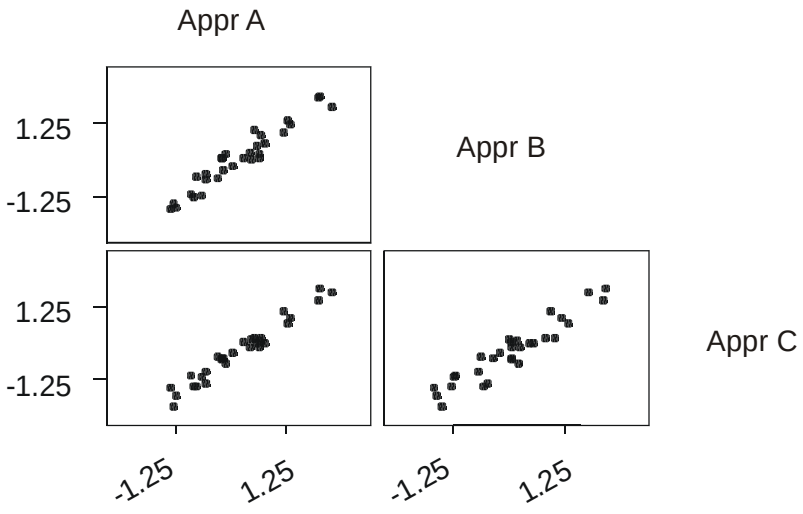


Figure III-B 14: Comparison X–Y Plots

Numerical Calculations

The Gage Repeatability and Reproducibility calculations are shown in Figures III-B 15 and III-B 16. Figure III-B 15 shows the data collection sheet on which all study results are recorded. Figure III-B 16 displays a report sheet on which all identifying information is to be recorded and all calculations made according to the prescribed formula.

Reproducible blank forms are available in the Sample Forms section. The procedure for doing the calculations after data have been collected is as follows:

(The following refers to Figure III-B 15)

1) Subtract the smallest reading from the largest reading in rows 1, 2 and 3; enter the result in row 5. Do the same for rows 6, 7, and 8; and 11, 12, and 13 and enter results in rows 10 and 15, respectively.
2) Entries in rows 5, 10 and 15 are ranges and therefore always positive values.
3) Total row 5 and divide the total by the number of parts sampled to obtain the average range for the first appraisers trials $\bar{R}_a$. Do the same for rows 10 and 15 to obtain $\bar{R}_b$ and $\bar{R}_c$.
4) Transfer the averages of rows 5, 10, and 15 ($\bar{R}_a$, $\bar{R}_b$, $\bar{R}_c$) to row 17. Add them together and divide by the number of appraisers and enter results $\bar{\bar{R}}$ (average of all ranges).
5) Enter $\bar{\bar{R}}$ (average value) in rows 19 and multiply by D_4 [49] to get the upper control limit. Note: D_4 is 3.27 if two trials are used. The value of the Upper Control Limit (UCL_R) of the individual ranges is entered in row 19. Note: The value of Lower Control Limit (LCL_R) for less than seven trials is equal to zero.
6) Repeat any readings that produced a range greater than the calculated UCL_R using the same appraiser and part as originally used, or discard those values and re-average and recompute $\bar{\bar{R}}$ and the limiting value UCL_R based upon the revised sample size. Correct the special cause that produced the out-of-control condition. If the data were plotted and analyzed using a control chart as discussed previously, this condition would have already been corrected and would not occur here.
7) Sum the rows (rows 1, 2, 3, 6, 7, 8, 11, 12, and 13). Divide the sum in each row by the number of parts sampled and enter these values in the right-most column labeled “Average”.

[49] See *Statistical Process Control (SPC) Reference Manual*, 2005, or other statistical reference source for a table of factors.

8) Add the averages in rows 1, 2 and 3 and divide the total by the number of trials and enter the value in row 4 in the $\bar{X}_a$ block. Repeat this for rows 6, 7 and 8; and 11, 12 and 13, and enter the results in the blocks for $\bar{X}_b$ and $\bar{X}_c$ in rows 9 and 14, respectively.

9) Enter the maximum and minimum averages of rows 4, 9 and 14 in the appropriate space in row 18 and determine the differences. Enter this difference in the space labeled $\bar{X}_{DIFF}$ in row 18.

10) Sum the measurements for each trial, for each part, and divide the total by the number of measurements (number of trials times the number of appraisers). Enter the results in row 16 in the spaces provided for part average.

11) Subtract the smallest part average from the largest part average and enter the result in the space labeled R_p in row 16. R_p is the range of part averages.

(The following refers to Figure III-B 16)

12) Transfer the calculated values of $\bar{\bar{R}}$, $\bar{X}_{DIFF}$ and R_p to the blanks provided on the report side of the form.

13) Perform the calculations under the column entitled “Measurement Unit Analysis” on the left side of the form.

14) Perform the calculations under the column entitled “% Total Variation” on the right side of the form.

15) Check the results to make sure no errors have been made.

Gage Repeatability and Reproducibility Data Collection Sheet

	Appraiser /Trial #	PART 1	2	3	4	5	6	7	8	9	10	AVERAGE
1	A 1	0.29	-0.56	1.34	0.47	-0.80	0.02	0.59	-0.31	2.26	-1.36	**0.194**
2	2	0.41	-0.68	1.17	0.50	-0.92	-0.11	0.75	-0.20	1.99	-1.25	**0.166**
3	3	0.64	-0.58	1.27	0.64	-0.84	-0.21	0.66	-0.17	2.01	-1.31	**0.211**
4	Average	**0.447**	**-0.607**	**1.260**	**0.537**	**-0.853**	**-0.100**	**0.667**	**-0.227**	**2.087**	**-1.307**	$\bar{X}_a$ = **0.1903**
5	Range	**0.35**	**0.12**	**0.17**	**0.17**	**0.12**	**0.23**	**0.16**	**0.14**	**0.27**	**0.11**	$\bar{R}_a$ = **0.184**
6	B 1	0.08	-0.47	1.19	0.01	-0.56	-0.20	0.47	-0.63	1.80	-1.68	**0.001**
7	2	0.25	-1.22	0.94	1.03	-1.20	0.22	0.55	0.08	2.12	-1.62	**0.115**
8	3	0.07	-0.68	1.34	0.20	-1.28	0.06	0.83	-0.34	2.19	-1.50	**0.089**
9	Average	**0.133**	**-0.790**	**1.157**	**0.413**	**-1.013**	**0.027**	**0.617**	**-0.297**	**2.037**	**-1.600**	$\bar{X}_b$ = **0.0683**
10	Range	**0.18**	**0.75**	**0.40**	**1.02**	**0.72**	**0.42**	**0.36**	**0.71**	**0.39**	**0.18**	$\bar{R}_b$ = **0.513**
11	C 1	0.04	-1.38	0.88	0.14	-1.46	-0.29	0.02	-0.46	1.77	-1.49	**-0.223**
12	2	-0.11	-1.13	1.09	0.20	-1.07	-0.67	0.01	-0.56	1.45	-1.77	**-0.256**
13	3	-0.15	-0.96	0.67	0.11	-1.45	-0.49	0.21	-0.49	1.87	-2.16	**-0.284**
14	Average	**-0.073**	**-1.157**	**0.880**	**0.150**	**-1.327**	**-0.483**	**0.080**	**-0.503**	**1.697**	**-1.807**	$\bar{X}_c$ = **-0.2543**
15	Range	**0.19**	**0.42**	**0.42**	**0.09**	**0.39**	**0.38**	**0.20**	**0.10**	**0.42**	**0.67**	$\bar{R}_c$ = **0.328**
16	Part Average	**0.169**	**-0.851**	**1.099**	**0.367**	**-1.064**	**-0.186**	**0.454**	**-0.342**	**1.940**	**-1.571**	$\bar{\bar{X}}$ = **.0014** R_p = **3.511**
17	([$\bar{R}_a$ = **0.184**] + [$\bar{R}_b$ = **0.513**] + [$\bar{R}_c$ = **0.328**]) / [# OF APPRAISERS = **3**] = **0.3417**											$\bar{\bar{R}}$ = **0.3417**
18	[Max $\bar{X}$ = **0.1903**] – [Min $\bar{X}$ = **-0.2543**] = $\bar{X}_{DIFF}$ = **0.4446**											
19	* [$\bar{\bar{R}}$ = **0.3417**] x [D_4 = **2.58**] = UCL_R = **0.8816**											

*D_4 = 3.27 for 2 trials and 2.58 for 3 trials. UCL_R represents the limit of individual R's. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and re-average and recompute $\bar{\bar{R}}$ and the limiting value from the remaining observations.

Notes:_____

Figure III-B 15: Completed GR&R Data Collection Sheet

Gage Repeatability and Reproducibility Report

Part No. & Name: Gage Name: Date:
Characteristics: Gage No: Performed by:
Specifications: Gage Type:

From data sheet: $\bar{\bar{R}}$ = **0.3417** $\bar{X}_{DIFF}$ = **0.4446** R_p = **3.511**

Measurement Unit Analysis

Repeatability – Equipment Variation (*EV*)

$EV = \bar{\bar{R}} \times K_1$

= **0.3417** × **0.5908**

= **0.20188**

Trials	K_1
2	0.8862
3	0.5908

Reproducibility – Appraiser Variation (*AV*)

$AV = \sqrt{(\bar{X}_{DIFF} \times K_2)^2 - (EV^2/(nr))}$

$= \sqrt{(0.4446 \times 0.5231)^2 - (0.20188^2/(10 \times 3))}$

= **0.22963**

n = parts *r = trials*

Appraisers	2	3
K_2	0.7071	0.5231

Repeatability & Reproducibility (*GRR*)

$GRR = \sqrt{EV^2 + AV^2}$

$= \sqrt{(0.20188^2 + 0.22963^2)}$

= **0.30575**

Parts	K_3
2	0.7071
3	0.5231
4	0.4467
5	0.4030
6	0.3742
7	0.3534
8	0.3375
9	0.3249
10	0.3146

Part Variation (*PV*)

$PV = R_p \times K_3$

= **1.10456**

Total Variation (*TV*)

$TV = \sqrt{GRR^2 + PV^2}$

$= \sqrt{(0.30575^2 + 1.10456^2)}$

= **1.14610**

% Total Variation (*TV*)

%EV = 100 [*EV/TV*]
= 100 [**0.20188**/**1.14610**]
= **17.62%**

%AV = 100 [*AV/TV*]
= 100 [**0.22963**/**1.14610**]
= **20.04%**

%GRR = 100 [*GRR/TV*]
= 100 [= **0.30575**/**1.14610**]
= **26.68%**

% PV = 100 [*PV/TV*]
= 100 [**1.10456**/**1.14610**]
= **96.38%**

ndc $= 1.41\left(PV/GRR\right)$
= 1.41(**1.10456/0.30575**)
= **5.094 ~ 5**

For information on the theory and constants used in the form see *MSA Reference Manual*, Fourth edition.

Figure III-B 16: Gage Repeatability and Reproducibility Report

Analysis of Results — Numerical

The *Gage Repeatability and Reproducibility Data Collection Sheet* and *Report* forms, Figures III-B 15 and III-B 16, will provide a method for the numerical analysis of the study data[50]. The analysis will estimate the variation and percent of process variation for the total measurement system and its components repeatability, reproducibility, and part variation. This information needs to be compared to and complement the results of the graphical analysis.

On the left side of the form (Figure III-B 16) under Measurement Unit Analysis, the standard deviation is calculated for each component of variation.

The repeatability or equipment variation (*EV* or σ_E) is determined by multiplying the average range ($\bar{\bar{R}}$) by a constant (K_1). K_1 depends upon the number of trials used in the gage study and is equal to the inverse of d_2^* which is obtained from Appendix C. d_2^* is dependent on the number of trials (m) and the number of parts times the number of appraisers (g) *(assumed to be greater than 15 for calculating the value of K_1)*

The reproducibility or appraiser variation (*AV* or σ_A) is determined by multiplying the maximum average appraiser difference ($\bar{X}_{DIFF}$) by a constant (K_2). K_2 depends upon the number of appraisers used in the gage study and is the inverse of d_2^* which is obtained from Appendix C. d_2^* is dependent on the number of appraisers (m) and $g = 1$, since there is only one range calculation. Since the appraiser variation is contaminated by the equipment variation, it must be adjusted by subtracting a fraction of the equipment variation. Therefore, the appraiser variation (*AV*) is calculated by

$$AV = \sqrt{\left(\bar{X}_{DIFF} \times K_2\right)^2 - \frac{(EV)^2}{nr}}$$

where n = number of parts and r = number of trials.

If a negative value is calculated under the square root sign, the appraiser variation (*AV*) defaults to zero.

The measurement system variation for repeatability and reproducibility (*GRR* or σ_M) is calculated by adding the square of the equipment variation and the square of the appraiser variation, and taking the square root as follows:

$$GRR = \sqrt{(EV)^2 + (AV)^2}$$

There are generally four different approaches to determine the process variation which is used to analyze the acceptability of the measurement variation:

[50] The numerical results in the example were developed as if they were computed manually; i.e., the results were carried and rounded to one additional decimal digit. Analysis by computer programs should maintain the intermediate values to the maximum precision of the computer/programming language. The results from a valid computer program may differ from the example results in the second or greater decimal place but the final analysis will remain the same.

1) using process variation
   - process variation, taken from the parts in the *GRR* study itself
   - use when the selected sample represents the expected process variation (preferred option)
2) surrogate process variation
   - use when sufficient samples to represent the process are not available but an existing process with similar process variation is available
3) Pp (or Ppk) target value
   - use when sufficient samples to represent the process are not available and an existing process with similar process variation is not available or the new process is expected to have less variability then an existing process
4) specification tolerance
   - When the measurement system is to be used to sort the process, and the process has a Pp <1.0

The part variation (part-to-part; part variation without measurement variation) (*PV* or σ_P) is determined by multiplying the range of part averages (R_p) by a constant (K_3). K_3 depends upon the number of parts used in the gage study and is the inverse of d_2^* which is obtained from Appendix C. d_2^* is dependent on the number of parts (*m*) and (*g*). In this situation $g = 1$ since there is only one range calculation.

The total variation (*TV* or σ_T) from the study is then calculated by summing the square of both the repeatability and reproducibility variation and the part variation (*PV*) and taking the square root as follows:

$$TV = \sqrt{(GRR)^2 + (PV)^2}$$

Using Historical Variation Information

To use this approach the information must be from a process that is in statistical control. If the process variation is known and its value is based on 6σ , then it can be used in place of the total study variation (*TV*) calculated from the gage study data. This is accomplished by performing the following two calculations:

1) $$TV = \frac{process\ variation}{6.00}$$

2) $$PV = \sqrt{(TV)^2 - (GRR)^2}$$

Using a Pp (or Ppk) target value

To use the Pp option, use the following *TV* in the *GRR* analysis:

$$since \;\; P_p = \frac{USL - LSL}{6\sigma_P} = \frac{USL - LSL}{6s} = \frac{USL - LSL}{6TV}$$

$$then \;\; TV = \frac{USL - LSL}{6Pp}$$

and $$PV = \sqrt{(TV)^2 - (GRR)^2}$$

Using the Tolerance (Specification Range).

When comparing measurement error from a *GRR* study to a tolerance, this is the same as comparing it to a production process with a Pp of 1.00. OEM customers rarely expect process variation to have as low a Pp(k) as 1.00, nor do they accept a process at that low of a performance level. It may make more sense to compare the measurement variation to a target production process performance level which meets the customer requirement.[51]

To use this option, use the following *TV* in the *GRR* analysis:

$$TV = \frac{USL - LSL}{6}$$

and $$PV = \sqrt{(TV)^2 - (GRR)^2}$$

Indices

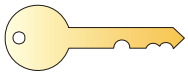
Once the variability for each factor in the gage study is determined, it can be compared to the total variation (*TV*). This is accomplished by performing the calculations on the right side of the gage report form (Figure III-B 16) under "% Total Variation."

The percent the equipment variation (*%EV*) consumes of the total variation (*TV*) is calculated by 100[*EV/TV*]. The percent that the other factors consume of the total variation can be similarly calculated as follows:

$$\%AV = 100\,[AV/TV]$$

$$\%GRR = 100\,[GRR/TV]$$

$$\%PV = 100\,[PV/TV]$$



THE SUM OF THE PERCENT CONSUMED BY EACH FACTOR WILL NOT EQUAL 100%.

The results of this percent total variation need to be evaluated to determine if the measurement system is acceptable for its intended application.

[51] for example, see Chrysler, Ford, and GM, *PPAP Manual*.

If the analysis is based on the tolerance instead of the process variation, then the Gage Repeatability and Reproducibility Report form (Figure III-B 16) can be modified so that the right-hand side of the form represents the percent of tolerance instead of percent of total variation. In that case, *%EV*, *%AV*, *%GRR* and *%PV* are calculated by substituting the value of tolerance *divided by six* in the denominator of the calculations in place of the total variation (*TV*). **Either or both approaches can be taken depending on the intended use of the measurement system and the desires of the customer.**

The final step in the numerical analysis is to determine the number of distinct categories that can be reliably distinguished by the measurement system. This is the number of non-overlapping 97% confidence intervals that will span the expected product variation.[52]

$$ndc = 1.41\left(PV/GRR\right)$$

Given that the graphical analysis has not indicated any special cause variation, the rule of thumb for gage repeatability and reproducibility (*%GRR*) may be found in Chapter II, Section D.

For analysis, the *ndc* is the maximum of one or the calculated value truncated to the integer. This result should be greater than or equal to 5.

To avoid a *ndc* = 0, which is possible with truncation alone, some computer programs will round up the calculated result. This can result in differences in final reports when the same data is evaluated by different programs

When using the Pp approach to *TV*, the calculation for ndc is:

$$TV^2 = PV^2 + GRR^2$$

$$or \quad PV^2 = TV^2 - GRR^2$$

then

$$ndc = 1.41\frac{PV}{GRR} = 1.41\frac{\sqrt{TV^2 - GRR^2}}{GRR}$$

Analysis of Variance (ANOVA) Method

Analysis of variance (ANOVA) is a standard statistical technique and can be used to analyze the measurement error and other sources of variability of data in a measurement systems study. In the analysis of variance, the variance can be decomposed into four categories: parts, appraisers, interaction between parts and appraisers, and replication error due to the gage.

[52] The importance of the number of distinct categories (*ndc*) in control and analysis activities is discussed in Chapter I, Section E, "Measurement Issues" (especially Figure I-E 3). Calculation of ndc is reviewed in Chapter II, Section B, "Analysis of Results - Numerical".

The advantages of ANOVA techniques as compared with Average and Range methods are:

- They are capable of handling any experimental set-up
- Can estimate the variances more accurately
- Extract more information (such as interaction between parts and appraisers effect) from the experimental data.

The disadvantages are that the numerical computations are more complex and users require a certain degree of statistical knowledge to interpret the results. The ANOVA method as described in the following sections is advised, especially if a computer is available.

Randomization and Statistical Independence

The method of collecting the data is important in an ANOVA method. If the data are not collected in a random manner, this can lead to a source of bias values. A simple way to assure a balanced design for (n) parts, (k) appraisers, and (r) trials is through randomization. One common approach to randomization is to write *A1* on a slip of paper to denote the measurement for the first appraiser on the first part. Do this up to $A(n)$ for the measurement by the first appraiser on the n^{th} part. Follow the same procedure for the next appraiser up to and including the k^{th} appraiser. The similar notation will be used where B_1, C_1 denotes the measurement for second and third appraiser on the first part. Once all nk combinations are written, then the slips of paper can be put in a hat or bowl. One at a time, a slip of paper is selected. These combinations (A_1, B_2, ...) are the measuring order in which the gage study will be performed. Once all nk combinations are selected, they are put back into the hat and the procedure is followed again. This is done for a total of r times to determine the order of experiments for each repeat.

> "The generation of random numbers is too important to be left to chance."
> Robert R. Covevou[56]

There are alternate approaches to generate a random sample. Care should be exercised to differentiate among random, haphazard and convenience sampling.[53]

In general, all efforts need to be taken to assure statistical independence within the study.

Conducting the Study

The data can be collected in a random manner using a form similar to Table III-B 6a. For our example, there are ten parts and three appraisers, and the experiment has been performed in random order three times for each part and appraiser combination.

[53] See Wheeler and Lyday, *Evaluating the Measurement Process*, Second Edition, 1989, p. 27.

[54] Robert R. Coveyou (1915 - 20 Feb 1996) was an American mathematician who was a health physicist with the Manhattan Project from 1943 during WW II. He became a recognized expert in pseudo-random number generators.

Graphical Analysis

Any of the graphical methods given in the discussion of Graphical Analysis above can be used in the graphical analysis of the data collected as part of an ANOVA study. These methods can be used to confirm and provide further insight to the data (i.e., trends, cycles, etc.).

One graphical method that is suggested is called an *interaction plot*. This plot confirms the results of the *F* test on whether or not the interaction is significant. In this particular interaction plot, the average measurement per appraiser per part vs. part number (1, 2, … etc.) is graphed in Figure III-B 17. The points for each appraiser average measurement per part are connected to form *k* (number of appraisers) lines. The way to interpret the graph is if the *k* lines are parallel there is no interaction term. When the lines are nonparallel, the interaction can be significant. The larger the angle of intersection is, the greater is the interaction. Appropriate measures should be taken to eliminate the causes for the interaction. In the example in Figure III-B 17, the lines are nearly parallel, indicating no significant interaction.

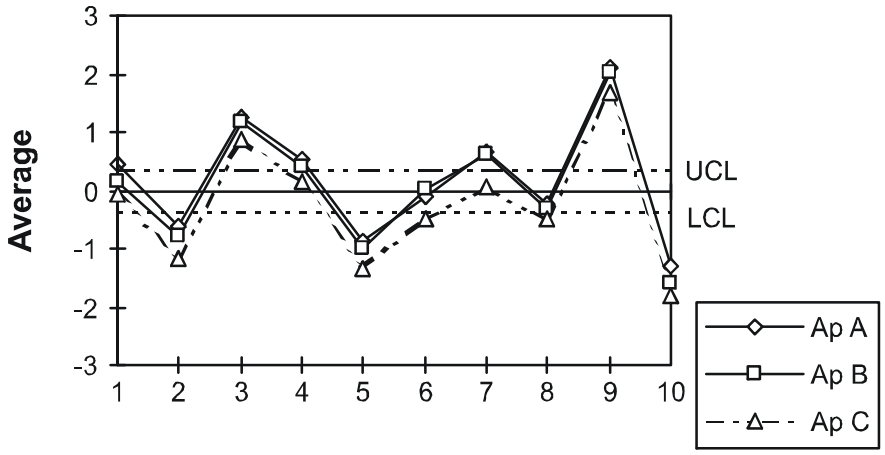


Figure III-B 17: Interaction Plot

Another graph sometimes of interest is the *residuals plot*. This graph is more a check for the validity of the assumptions. This assumption is that the gage (error) is a random variable from a normal distribution. The residuals, which are the differences between the observed readings and predicted values, are plotted. Predicted value is the average of the repeated readings for each appraiser for each part. If the residuals are not randomly scattered above and below zero (horizontal reference line), it could be because the assumptions are incorrect and further investigation of the data is suggested.

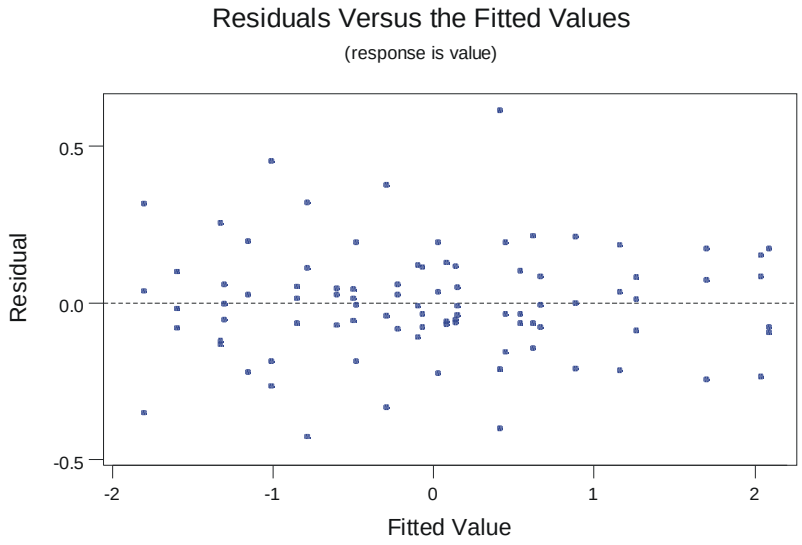


Figure III-B 18: Residual Plot

Numerical Calculations

Although the values can be calculated manually, most people will use a computer program to generate what is called the Analysis of Variance (ANOVA) table (see Appendix A).

The ANOVA table here is composed of five columns (see Table III-B 7).

- *Source* column is the cause of variation.
- *DF* column is the *degree of freedom* associated with the source.
- *SS* or *sum of squares* column is the deviation around the mean of the source.
- *MS* or *mean square* column is the sum of squares divided by degrees of freedom.
- *F-ratio* column, calculated to determine the statistical significance of the source value.

The ANOVA table is used to decompose the total variation into four components: parts, appraisers, interaction of appraisers and parts, and repeatability due to the instrument.

For analysis purposes, negative variance components are set to zero.

This information is used to determine the measurement system characteristics as in the Average and Range Method

Source	DF	SS	MS	F
Appraiser	2	3.1673	1.58363	34.44*
Parts	9	88.3619	9.81799	213.52*
Appraiser by Part	18	0.3590	0.01994	0.434
Equipment	60	2.7589	0.04598	
Total	89	94.6471		

* Significant at $\alpha = 0.05$ level

Table III-B 7: ANOVA Table

Table III-B 7 shows the ANOVA calculations for the example data from Figure III-B 15 assuming a balanced two-factor factorial design. Both factors are considered to be random. Table III-B 9 shows the comparison of the ANOVA method with the Average and Range method. Table III-B 10 shows the *GRR* report for the ANOVA method.

Estimate of Variance	Std. Dev. (σ)	% Total Variation	% Contribution
$\tau^2 = 0.039973$ (Repeatability)	$EV = 0.199933$	18.4	3.4
$\omega^2 = 0.051455$ (Appraiser)	$AV = 0.226838$	20.9	4.4
$\gamma^2 = 0$ (Interaction)	$INT = 0$	0	0
System = 0.09143 ($\tau^2 + \gamma^2 + \omega^2$)	$GRR = 0.302373$	27.9	7.8
$\sigma^2 = 1.086446$ (Part)	$PV = 1.042327$	96.0	92.2
Total Variation	$TV = 1.085$	100.0	

Table III-B 8: ANOVA Analysis % Variation & Contribution
(Estimate of variance is based on model without interaction)

$$ndc = 1.41\left(1.04233 \big/ .30237\right) = 4.861 \cong 5$$

$$Total\ Variation\ (TV) = \sqrt{GRR^2 + PV^2}$$

$$\%\ of\ TotalVariation = 100\left(\frac{\sigma_{(components)}}{\sigma_{(total)}}\right)$$

$$\%\ Contribution\ (to\ Total\ Variance) = 100\left(\frac{\sigma^2_{(components)}}{\sigma^2_{(total)}}\right)$$

Analysis of *GRR* Studies

Both the Average and Range method and ANOVA method will provide information concerning the causes of measurement system or gage variation.

For example, if repeatability is large compared to reproducibility, the reasons may be:

- The instrument needs maintenance.
- The gage may need to be redesigned to be more rigid.
- The clamping or location for gaging needs to be improved.
- There is excessive within-part variation.

If reproducibility is large compared to repeatability, then possible causes could be:

- The appraiser needs to be better trained in how to use and read the gage instrument.
- Calibrations on the gage dial are not clear.

A fixture of some sort may be needed to help the appraiser use the gage more consistently.

Method	Lower 90% *CL*[55]	Std. Dev.	Upper 90% *CL*	% of Total Variation
*GRR**				
EV	0.175	.202	0.240	17.6
AV	0.133	.230	1.016	20.1
INTERACTION	--	na	--	na
GRR	0.266	.306	0.363	26.7
PV		1.104		96.4
ANOVA				
EV	0.177	0.200	0.231	18.4
AV	0.129	0.227	1.001	20.9
INTERACTION	--	0	--	0
GRR	0.237	0.302	1.033	27.9
PV		1.042		96.0

* In the average and range method, the interaction component cannot be estimated.

Table III-B 9: Comparison of ANOVA and Average and Range Methods

Part No. & Name: Gage Name: Date:
Characteristics: Gage No: Performed by:
Specifications: Gage Type:

	STD. DEV.	% TOTAL VARIATION	PERCENT CONTRIBUTION
Repeatability (*EV*)	0.200	18.4	3.4
Reproducibility (*AV*)	0.227	20.9	4.4
Appraiser by Part (*INT*)	0	0	0
GRR	0.302	27.9	7.9
Part (*PV*)	1.042	96.0	92.2

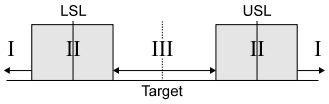
Measurement System is acceptable for Process Control and Analysis.

Note:
Tolerance = N.A. Total variation (*TV*) = 1.085
Number of distinct data categories (*ndc*) = 4

Table III-B 10: *GRR* ANOVA Method Report

[55] *CL* = Confidence Limit

Section C
Attribute Measurement Systems Study



Attribute measurement systems are the class of measurement systems where the measurement value is one of a finite number of categories. This is contrasted to the variables measurement system which can result in a continuum of values. The most common of these is a go/no-go gage which has only two possible results. Other attribute systems, for example visual standards, may result in five to seven classifications, such as very good, good, fair, poor, very poor. The analyses described in the preceding chapters cannot be used to evaluate such systems.

As discussed in Chapter I, Section B, there is a quantifiable risk when using any measurement systems in making decisions. Since the largest risk is at the category boundaries, the most appropriate analysis would be the quantification of the measurement system variation with a gage performance curve. (See Chapter IV Section F)

Risk Analysis Methods

In some attribute situations it is not feasible to get sufficient parts with variable reference values. In such cases, the risks of making wrong or inconsistent[56] decisions can be evaluated by using:

- Hypothesis Test Analyses
- Signal Detection Theory

Since these methods do not quantify the measurement system variability, they should be used only with the consent of the customer. Selection and use of such techniques should be based on good statistical practices, an understanding of the potential sources of variation which can affect the product and measurement processes, and the effect of an incorrect decision on the remaining processes and the final customer.

The sources of variation of attribute systems should be minimized by using the results of human factors and ergonomic research.

Possible Approaches

Scenario

The production process is in statistical control and has the performance indices of $Pp = Ppk = 0.5$ which is unacceptable. Because the process is producing nonconforming product, a containment action is required to cull the unacceptable parts from the production stream.

[56] This includes the comparison of multiple appraisers.

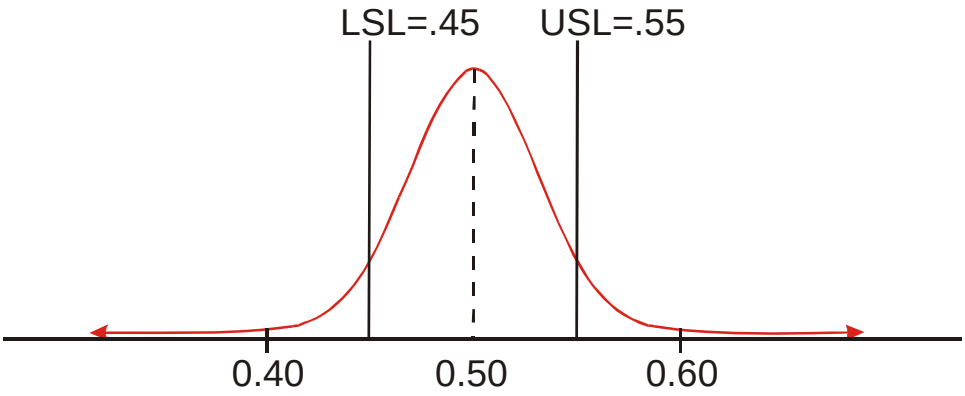


Figure III-C 1: Example Process with *Pp* = *Ppk* = 0.50

For the containment activity the process team selected an attribute gage that compares each part to a specific set of limits and accepts the part if the limits are satisfied; otherwise it rejects the part (known as a go/no-go gage). Most gages of this type are set up to accept and reject based on a set of master parts. Unlike a variable gage, this attribute gage cannot indicate how good or how bad a part is, but only that the part is accepted or rejected (i.e., 2 categories). As for all gages, this attribute gage will have "Gray" areas where wrong decisions can be made (see Figure III-C 2 below and Chapter II, Section B).

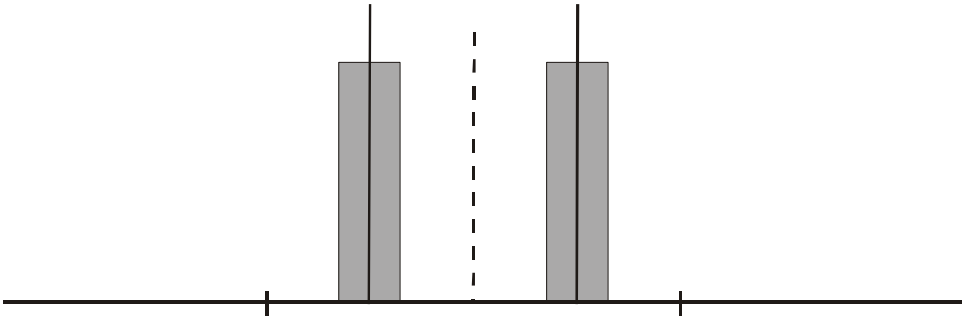


Figure III-C 2: The "Gray" Areas Associated with the Measurement System

Since this has not yet been documented by the team, it needs to study the measurement system. However, to address the areas of risk around the specification limits, the team chose approximately 25% of the parts at or close to the lower specification limit and 25% of the parts at or close to the upper specification limit. In some cases where it is difficult to make such parts the team may decide to use a lower percentage recognizing that this may increase the variability of the results. If it is not possible to make parts close to the specification limits the team should reconsider the use of attribute gaging for this process. As appropriate for each characteristic, the parts should be independently measured with a variable gage with acceptable variation (e.g., a CMM). When measuring a true attribute that cannot be

measured with a variable gauge use other means such as experts to pre-determine which samples are good or defective.

Three appraisers are used, with each appraiser making three decisions on the each part.

An acceptable decision was designated with a one (1) and an unacceptable decision with zero (0). The reference decision and variable reference values shown in Table III-C 1 were not initially determined. The table also indicates in the "Coded" column whether the part is area I, area II, or area III part indicated by "– ", "x", and "+" respectively.

Hypothesis Test Analyses – Cross-Tab Method

Since the team did not know the reference decisions for the parts, they developed cross-tabulations comparing each of the appraisers to the other.

The cross-tabulation process analyzes distribution data for two or more categorical variables. The results – presented in a matrix format – form a contingency table that illustrates the interdependency between variables.[57]

[57] Cross-tabulation is available in many statistical analysis software packages and is used in spreadsheet pivot table functions.

Part	A - 1	A - 2	A - 3	B - 1	B - 2	B - 3	C - 1	C - 2	C - 3	Reference	Ref Value	Code
1	1	1	1	1	1	1	1	1	1	1	0.476901	+
2	1	1	1	1	1	1	1	1	1	1	0.509015	+
3	0	0	0	0	0	0	0	0	0	0	0.576459	–
4	0	0	0	0	0	0	0	0	0	0	0.566152	–
5	0	0	0	0	0	0	0	0	0	0	0.570360	–
6	1	1	0	1	1	0	1	0	0	1	0.544951	x
7	1	1	1	1	1	1	1	0	1	1	0.465454	x
8	1	1	1	1	1	1	1	1	1	1	0.502295	+
9	0	0	0	0	0	0	0	0	0	0	0.437817	–
10	1	1	1	1	1	1	1	1	1	1	0.515573	+
11	1	1	1	1	1	1	1	1	1	1	0.488905	+
12	0	0	0	0	0	0	0	1	0	0	0.559918	x
13	1	1	1	1	1	1	1	1	1	1	0.542704	+
14	1	1	0	1	1	1	1	0	0	1	0.454518	x
15	1	1	1	1	1	1	1	1	1	1	0.517377	+
16	1	1	1	1	1	1	1	1	1	1	0.531939	+
17	1	1	1	1	1	1	1	1	1	1	0.519694	+
18	1	1	1	1	1	1	1	1	1	1	0.484167	+
19	1	1	1	1	1	1	1	1	1	1	0.520496	+
20	1	1	1	1	1	1	1	1	1	1	0.477236	+
21	1	1	0	1	0	1	0	1	0	1	0.452310	x
22	0	0	1	0	1	0	1	1	0	0	0.545604	x
23	1	1	1	1	1	1	1	1	1	1	0.529065	+
24	1	1	1	1	1	1	1	1	1	1	0.514192	+
25	0	0	0	0	0	0	0	0	0	0	0.599581	–
26	0	1	0	0	0	0	0	0	1	0	0.547204	x
27	1	1	1	1	1	1	1	1	1	1	0.502436	+
28	1	1	1	1	1	1	1	1	1	1	0.521642	+
29	1	1	1	1	1	1	1	1	1	1	0.523754	+
30	0	0	0	0	0	1	0	0	0	0	0.561457	x
31	1	1	1	1	1	1	1	1	1	1	0.503091	+
32	1	1	1	1	1	1	1	1	1	1	0.505850	+
33	1	1	1	1	1	1	1	1	1	1	0.487613	+
34	0	0	1	0	0	1	0	1	1	0	0.449696	x
35	1	1	1	1	1	1	1	1	1	1	0.498698	+
36	1	1	0	1	1	1	1	0	1	1	0.543077	x
37	0	0	0	0	0	0	0	0	0	0	0.409238	–
38	1	1	1	1	1	1	1	1	1	1	0.488184	+
39	0	0	0	0	0	0	0	0	0	0	0.427687	–
40	1	1	1	1	1	1	1	1	1	1	0.501132	+
41	1	1	1	1	1	1	1	1	1	1	0.513779	+
42	0	0	0	0	0	0	0	0	0	0	0.566575	–
43	1	0	1	1	1	1	1	1	0	1	0.462410	x
44	1	1	1	1	1	1	1	1	1	1	0.470832	+
45	0	0	0	0	0	0	0	0	0	0	0.412453	–
46	1	1	1	1	1	1	1	1	1	1	0.493441	+
47	1	1	1	1	1	1	1	1	1	1	0.486379	+
48	0	0	0	0	0	0	0	0	0	0	0.587893	–
49	1	1	1	1	1	1	1	1	1	1	0.483803	+
50	0	0	0	0	0	0	0	0	0	0	0.446697	–

Table III-C 1: Attribute Study Data Set

The first step is to summarize the observed data. Reviewing Table III-C 1, the team examined the data for pairs of observers, counting when they agreed and when they disagreed for each set of evaluations. That is, for the evaluations, there are 34 times where A-1 = 1 and B-1 = 1; there are 32 times where A-2 = 1 and B-2 = 1; and there are 31 times where A-3 = 1 and B-3 = 1for a total agreement of 97. The table below was constructed to summarize the data distribution for the observer pair A*B. Similar tables were prepared for observer pairs B*C and A*C.

		B		
		.00	1.00	Total
A	.00	44 (agree)	6 (disagree)	50
	1.00	3 (disagree)	97 (agree)	100
Total		47	103	150

The second step is to estimate the expected data distribution. What is the probability that an observer pair will agree or disagree on an observation purely by chance? In 150 observations Observer A rejected the part 50 times and Observer B rejected the part 47 times:

$p_{A0} = 47/150 = 0.313$

$p_{B0} = 50/150 = 0.333$

Since the two observers are independent, the probability that they agree that the part is bad is given by:

$$p(A0 \cap B0) = p_{A0}\, p_{B0} = 0.104$$

The expected number of times for Observer A and Observer B agree the part is bad is estimated by multiplying the combined probability by the number of observations:

$$150 \times (p_{A0}\, p_{B0}) = 150 \times (47/150) \times (50/150) = 15.7$$

The team made similar estimations of each category pair for each observer pair to complete the following tables:

A * B Crosstabulation

			B		
			.00	1.00	Total
A	.00	Count	44	6	50
		Expected Count	15.7	34.3	50.0
	1.00	Count	3	97	100
		Expected Count	31.3	68.7	100.0
Total		Count	47	103	150
		Expected Count	47.0	103.0	150.0

B * C Crosstabulation

			C		
			.00	1.00	Total
B	.00	Count	42	5	47
		Expected Count	16.0	31.0	47.0
	1.00	Count	9	94	103
		Expected Count	35.0	68.0	103.0
Total		Count	51	99	150
		Expected Count	51.0	99.0	150.0

A * C Crosstabulation

			C		
			.00	1.00	Total
A	.00	Count	43	7	50
		Expected Count	17.0	33.0	50.0
	1.00	Count	8	92	100
		Expected Count	34.0	66.0	100.0
Total		Count	51	99	150
		Expected Count	51.0	99.0	150.0

Table III-C 2: Cross tabulation Study Results

To determine the level of this agreement the team uses the (Cohen's) *kappa* which measures the agreement between the evaluations of two raters when both are rating the same object. A value of 1 indicates perfect agreement. A value of 0 indicates that agreement is no better than chance. *Kappa* is only available for tables in which both variables use the same category values and both variables have the same number of categories.[58]

[58] There are a number of statistics which can be used to determine inter-rater agreement. Different statistics are appropriate for different types of measurement. See Reference List including: Bland, J. M., and Altman, D. G. (1986); Cohen, J. (1960); Everitt, B. (1996); Fleiss, J. L. (1971); Krippendorff, K. (2004); Saal, F.E., Downey, R.G. and Lahey, M.A. (1980); Shrout, P. and Fleiss, J. L. (1979); and Uebersax, John S. (1987)

Kappa is a measure of *interrater agreement* that tests if the counts in the diagonal cells (the parts that receive the same rating) differ from those expected by chance alone.

Let p_o = the sum of the observed proportions in the diagonal cells

p_e = the sum of the expected proportion in the diagonal cells

then

$$kappa = \frac{p_o - p_e}{1 - p_e}$$

Kappa is a measure rather than a test.[59] Its size is judged by using an asymptotic standard error to construct a *t* statistic. A general rule of thumb is that values of *kappa* greater than 0.75 indicate good to excellent agreement (with a maximum *kappa* = 1); values less than 0.40 indicate poor agreement.

> *Kappa* takes no account of the size of disagreement between the raters, but only whether they agree or not.[60]

Upon calculating the *kappa* measures for the appraisers, the team came up with the following:

Kappa	A	B	C
A	–	.86	.78
B	.86	–	.79
C	.78	.79	–

Table III-C 3: Kappa Summary

This analysis indicates that all the appraisers show good agreement between each other.

This analysis is necessary to determine if there are any differences among the appraisers but it does not tell us how well the measurement system sorts good parts from bad. For this analysis the team had the parts evaluated using a variable measurement system and used the results to determine the reference decision.

With this new information another group of cross-tabulations was developed comparing each appraiser to the reference decision.

[59] As in all such categorical evaluations, a large number of parts covering the entire spectrum of possibilities is necessary.

[60] When the observations are measured on an ordinal categorical scale a *weighted kappa* can be used to better measure agreement. Agreement between the two raters is treated as for *kappa* but disagreements are measured by the number of categories by which the raters differ.

A * REF Crosstabulation

			REF		
			.00	1.00	Total
A	.00	Count	45	5	50
		Expected Count	16.0	34.0	50.0
	1.00	Count	3	97	100
		Expected Count	32.0	68.0	100.0
Total		Count	48	102	150
		Expected Count	48.0	102.0	150.0

B * REF Crosstabulation

			REF		
			.00	1.00	Total
B	.00	Count	45	2	47
		Expected Count	15.0	32.0	47.0
	1.00	Count	3	100	103
		Expected Count	33.0	70.0	103.0
Total		Count	48	102	150
		Expected Count	48.0	102.0	150.0

C * REF Crosstabulation

			REF		
			.00	1.00	Total
C	.00	Count	42	9	51
		Expected Count	16.3	34.7	51.0
	1.00	Count	6	93	99
		Expected Count	31.7	67.3	99.0
Total		Count	48	102	150
		Expected Count	48.0	102.0	150.0

Table III-C 4: Comparisons of Appraisers to Reference

The team also calculated the *kappa* measure to determine the agreement of each appraiser to the reference decision:

	A	B	C
kappa	.88	.92	.77

These values can be interpreted as each of the appraisers has good agreement with the standard.

The process team then calculated the effectiveness of the measurement system.

$$Effectiveness = \frac{number\ of\ correct\ decisions}{total\ opportunities\ for\ a\ decision}$$

	% Appraiser[1]			% Score vs. Attribute[2]		
Source	Appraiser A	Appraiser B	Appraiser C	Appraiser A	Appraiser B	Appraiser C
Total Inspected	50	50	50	50	50	50
# Matched	42	45	40	42	45	40
False Negative (appraiser biased toward rejection)				0	0	0
False Positive (appraiser biased toward acceptance)				0	0	0
Mixed				8	5	10
95% UCI	93%	97%	90%	93%	97%	90%
Calculated Score	**84%**	**90%**	**80%**	**84%**	**90%**	**80%**
95% LCI	71%	78%	66%	71%	78%	66%

	System % Effective Score[3]	System % Effective Score vs. Reference[4]
Total Inspected	50	50
# in Agreement	39	39
95% UCI	89%	89%
Calculated Score	**78%**	**78%**
95% LCI	64%	64%

Notes

(1) Appraiser agrees with him/herself on all trials

(2) Appraiser agrees on all trials with the known standard

(3) All appraisers agreed within and between themselves

(4) All appraisers agreed within & between themselves AND agreed with the reference

(5)UCI and LCI are the upper and lower confidence interval bounds, respectively

Table III-C 5: Study Effectiveness Table

Multiple tests of hypothesis between each pair of appraisers can be conducted with the null hypothesis:

H_0: The effectiveness of both appraisers is the same

Since the calculated score of each appraiser falls within the confidence interval of the other, the team concludes that they cannot reject the null hypotheses. This reinforces the conclusions from the *kappa* measures.

For further analysis, one of the team members brought out the following table which provides guidelines for each appraiser's results:

Decision Measurement system	Effectiveness	Miss Rate	False Alarm Rate
Acceptable for the appraiser	≥ 90%	≤ 2%	≤ 5%
Marginally acceptable for the appraiser – may need improvement	≥80%	≤ 5%	≤ 10%
Unacceptable for the appraiser – needs improvement	< 80%	> 5%	> 10%

Table III-C 6: Example Effectiveness Criteria Guidelines

Summarizing all the information they already had, the team came up with this table:

	Effectiveness	Miss Rate	False Alarm Rate
A	84%	6.3%	4.9%
B	90%	6.3%	2.0%
C	80%	12.5%	8.8%

Table III-C 7: Study Effectiveness Summary

These results showed that the measurement system had different levels of performance in Effectiveness, Miss Rate and False Alarm Rate depending on the appraiser. No single appraiser had acceptable results in all three categories. No single appraiser had unacceptable results in all three categories. Do the acceptance guidelines need to be changed for this process? Are these risks acceptable? Do the appraisers need better training? Could the testing environment be improved? Most importantly, what did the customer think about the measurement system and these results – what were their expectations?

Does the customer accept these risk levels?

Sample Size

The question always arises: "How many samples should be used in the study?" To the dismay of most people, the answer is "enough". The purpose of any measurement study (variables or attribute) is to understand the properties of the measurement system.. A sufficient number of samples should be selected to cover the expected operating range (see also Chapter II Section C). With attribute measurement systems, the area of interest are the

Type II areas (see Chapter I, Section B). If the inherent process capability is good (i.e., large C_p, C_{pk} or P_p, P_{pk}) then a small random sample may not have many (or any) samples in this area. This means that as the process capability improves , the required random sample for the attribute study should become larger).

In the example above, the indices were P_p, P_{pk} = 0.5 (i.e. an expected process performance of approximately 13% nonconformance), the sample selected was 50.

An alternate approach to large samples is a "salted sample" where parts are selected specifically from the Type II areas to augment a random sample to ensure that the effect of appraiser variability is seen.

Concerns

1) There are no theory-based decision criteria on acceptable risk. The above guidelines are heuristic and developed based on individual "beliefs" about what will pass as "acceptable". The final decision criteria should be based on the impact (i.e., risk) to the remaining process and final customer. This is a subject matter decision – not a statistical one.

2) The analysis above is data dependent. For example, if the process indices were *Pp* = *Ppk* = 1.33, then all the decisions would be correct since no parts would fall in the region II (the "gray" areas) of the measurement system.

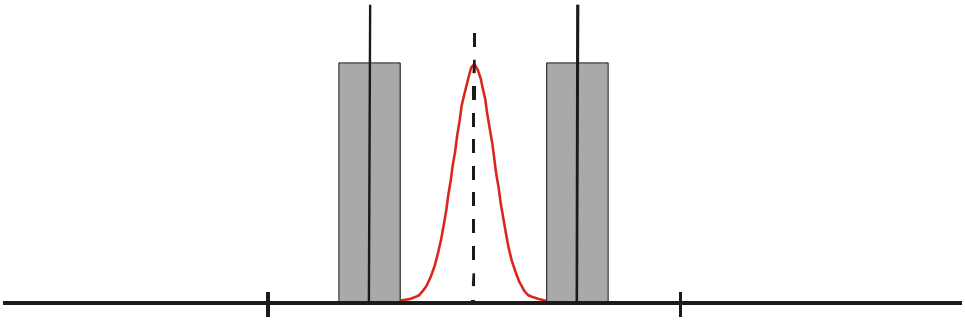


Figure III-C 3: Example Process with *Pp* = *Ppk* = 1.33

With this new situation, it would be concluded that all the appraisers were acceptable since there would be no decision errors.

3) There is usually a misconception of what the cross-tabulation results really mean. For example, appraiser B results from Table III-C 4 are:

B * REF Crosstabulation

			REF		
			.00	1.00	Total
B	.00	Count	45	2	47
		% within REF	93.8%	2.0%	31.3%
	1.00	Count	3	100	103
		% within REF	6.3%	98.0%	68.7%
Total		Count	48	102	150
		% within REF	100.0%	100.0%	100.0%

Since the purpose of the inspection is to find all the nonconforming parts, most people look at the upper left corner as a measure of the effectiveness of finding bad parts. This percent is the probability of saying a part is bad given that it is bad:

$$\Pr(\textit{calling the part bad} \mid \textit{a bad part})$$

Assuming that the process is improved to *Pp* = *Ppk* = 1.00, the probability of interest to the producer is:

$$\Pr\left(\textit{the part is bad} \mid \textit{it is called bad}\right)$$

To determine this from the above data, Bayes' Theorem must be used.

$$\Pr\left(\textit{bad} \mid \textit{called bad}\right) = \frac{\Pr\left(\textit{called bad} \mid \textit{bad}\right) * \Pr\left(\textit{bad}\right)}{\Pr\left(\textit{called bad} \mid \textit{bad}\right) * \Pr\left(\textit{bad}\right) + \Pr\left(\textit{called bad} \mid \textit{good}\right) * \Pr\left(\textit{good}\right)}$$

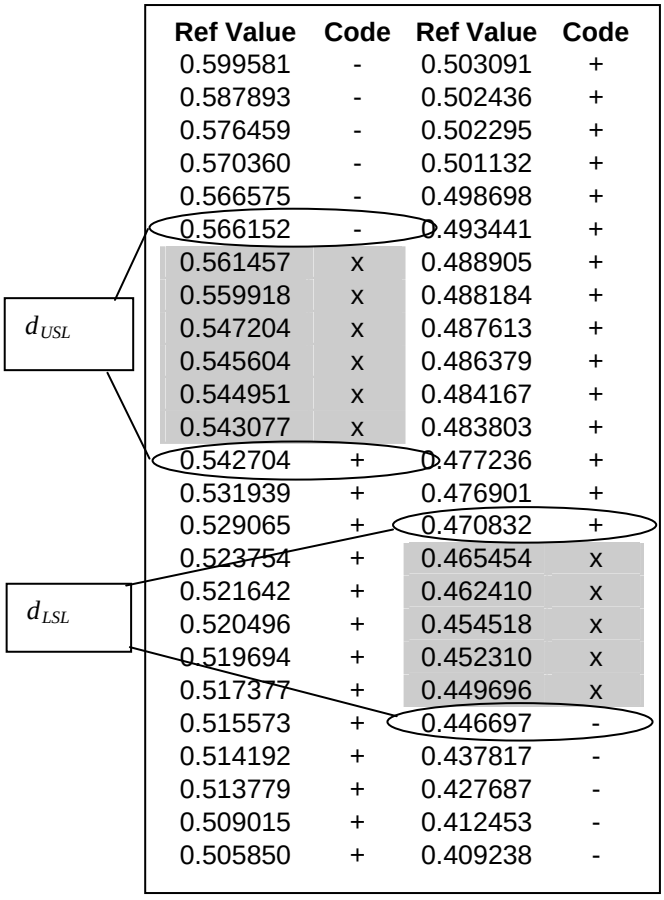
$$\Pr\left(\textit{bad} \mid \textit{called bad}\right) = \frac{.938 * (.0027)}{.938 * (.0027) + .020 * (.9973)}$$

$$\Pr\left(\textit{bad} \mid \textit{called bad}\right) = .11$$

That is, these results indicate that if the part is called bad there is only a 1 out of 10 chance that it is truly bad.

4) The analysis does not use the variable data information or even the relative order information which was available when the reference decision values were determined.

Signal Detection Approach



Ref Value	Code	Ref Value	Code
0.599581	-	0.503091	+
0.587893	-	0.502436	+
0.576459	-	0.502295	+
0.570360	-	0.501132	+
0.566575	-	0.498698	+
0.566152	-	0.493441	+
0.561457	x	0.488905	+
0.559918	x	0.488184	+
0.547204	x	0.487613	+
0.545604	x	0.486379	+
0.544951	x	0.484167	+
0.543077	x	0.483803	+
0.542704	+	0.477236	+
0.531939	+	0.476901	+
0.529065	+	0.470832	+
0.523754	+	0.465454	x
0.521642	+	0.462410	x
0.520496	+	0.454518	x
0.519694	+	0.452310	x
0.517377	+	0.449696	x
0.515573	+	0.446697	-
0.514192	+	0.437817	-
0.513779	+	0.427687	-
0.509015	+	0.412453	-
0.505850	+	0.409238	-

Table III-C 8: Table III-C 1 sorted by Ref Value

An alternate approach is to use Signal Detection theory[61] to determine an approximation of the width of the region II area and from this, the measurement system *GRR*. This requires that each of the sample parts can be evaluated offline by a variables measurement system. This reference value is shown in the column *Ref Value* in Table III-C 1.

Steps:

1. Determine the Tolerance (specification range); from Figure III-C 1:
   USL = .550
   LSL = .450

   Then the Tolerance = USL – LSL = .100 . This value will be used to calculate the *GRR*.

Guidelines:

- If Ppk>1, compare measurement system to process.
- If Ppk<1, compare measurement system to tolerance.

This "rule" amounts to comparing the measurement system to whichever is most restrictive, the process or the tolerance.

In this example, the Ppk = .5 (see Figure III-C 1), so the process is greater than the tolerance and this measurement system should therefore be compared to tolerance.

For the data in Table III-C 1.

2) Rank order the data (and adjacent cells) from highest to lowest based on the individual Reference Values (see Table III-C 8; Note: this table has been divided into two columns to save space.).

3) Identify the beginning and end points of the two area IIs. In Table III-C 8 this is shown by the column *Code*:

   + = accepted with total agreement

   – = rejected with total agreement (in Table III-C 1)

   x = disagreement

   The width of these zones is what we are trying to determine, and the average width of these zones will be used to compare the

[61] See Reference List: Baker 1975

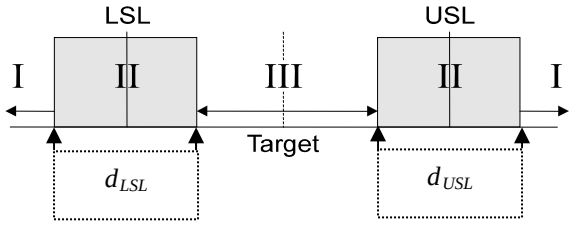
measurement system to the specification tolerance, or to the process 6 sigma width (depending on the situation).

4) Referring to the above graphic, let d_{USL} = distance between the last part accepted by all appraisers to the first part rejected by all. This distance is equivalent to the gray zone II surrounding the USL above, bounded by the upper zone I to the right, and Zone III to the left. Note the values circled in Table III-C 8 above.

Zone I = parts agreed by all appraisers to be rejected.

Zone III = parts agreed by all appraisers to be accepted.

Zone II = questionable parts without 100% agreement, surrounding each specification limit.



5) Let d_{LSL} = distance between the last part accepted by all appraisers in Zone III to the first part rejected by all appraisers in Zone I.

Let d_i = distance between the last part accepted by all appraisers to the first part rejected by all (for each specification).

Then,

d = average (d_i)

is an estimate[62] of the width of region II areas and, thus, an estimate of the $GRR = 6*\sigma_{GRR}$

In this example (p. 126) where the tolerance is 0.100,

d_{LSL} = 0.470832 – 0.446697 = 0.024135

d_{USL} = 0.566152 – 0.542704 = 0.023448

d = 0.0237915

or the estimated *%GRR* is,

%GRR = 24%

Since this example was generated by a measurement system with an actual *%GRR* = 25%, this estimate will lead to the same evaluation of the measurement system.

If only ordered data information is available, this technique can still be used but requires subject matter knowledge to quantify the *d*'s.

[62] The "goodness" of this estimate depends on the sample size and how close the sample represents the process. The larger the sample, the better the estimate.

Analytic Method[63]

As with any measurement system, the stability of the process should be verified and, if necessary, monitored. For attribute measurement systems, attribute control charting of a constant sample over time is a common way of verifying stability[64].

For an attribute measurement system, the concept of the Gage Performance Curve (see Chapter IV, Section F) is used for developing a measurement system study, which is used to assess the amount of repeatability and bias of the measurement system. This analysis can be used on both single and double limit measurement systems. For a double limit measurement system, only one limit need be examined with the assumptions of linearity and uniformity of error. For convenience, the lower limit will be used for discussion.

In general, the attribute measurement system study consists of obtaining the reference values for several selected parts. These parts are evaluated a number of times, (m), with the total number of accepts (a), for each part being recorded. From the results, repeatability and bias can be assessed.

The first stage of the attribute study is the part selection. It is essential that the reference value be known for each part used in the study. Eight parts should be selected at as nearly equidistant intervals as practical. The maximum and minimum values should represent the process range. Although this selection does not affect confidence in the results, it does affect the total number of parts needed to complete the gage study. The eight parts must be run through the gage, $m = 20$ times, and the number of accepts, (a), recorded.

For the total study, the smallest part must have the value $a = 0$; the largest part, $a = 20$; and the six other parts, $1 \leq a \leq 19$. If these criteria are not satisfied, more parts with known reference values, (X), must be run through the gage until the above conditions are met. If, for the smallest value $a \neq 0$, then smaller and smaller parts are taken and evaluated until $a = 0$. If, for the largest value $a \neq 20$, then larger and larger parts are taken until $a = 20$. If six of the parts do not have $1 \leq a \leq 19$, additional parts can be taken at selected points throughout the range. These points are taken at the midpoints of the part measurements already measured in the study. The first interval at the $a = 0$ end starts from the largest measurement where $a = 0$. For the $a = 20$ end, the first interval starts at the smallest measurement where $a = 20$. For best results, samples should be taken at both the $a = 0$ and $a = 20$ ends and worked toward the middle of the part range. If necessary, the procedure can be repeated until the criteria are met.

Once the data collection criteria have been satisfied, the probabilities of acceptance must be calculated for each part using the following equations:

[63] Adapted with permission from "Analysis of Attribute Gage Systems" by J. McCaslin & G. Gruska, ASQC, 1976.

[64] Caveat: np > 4

$$P_a^{'} = \begin{cases} \dfrac{a + 0.5}{m} & if\ \dfrac{a}{m} < 0.5, \quad a \neq 0 \\ \dfrac{a - 0.5}{m} & if\ \dfrac{a}{m} > 0.5 \quad a \neq 20 \\ 0.5 & if\ \dfrac{a}{m} = 0.5 \end{cases}$$

The adjustments cover the instances where $1 \leq a \leq 19$. For the instances where $a = 0$ set $P_a' = 0$ except for the largest reference value with $a = 0$, in which $P_a' = 0.025$. For the instances where $a = 20$ then $P_a' = 1$ except for the smallest reference value with $a = 20$ in which $P_a' = 0.975$.

Once the P_a' has been calculated for each X_T, the Gage Performance Curve (*GPC*) can be developed. Although the *GPC* can be presented graphically (see Figure III-C 5), use of normal probability paper (see Figure III-C 4) yields more accurate estimates of the repeatability and bias.

The calculated probabilities are plotted on normal probability paper and a line of best fit is drawn through these points. The **bias** is equal to the lower limit minus the reference value measurement that corresponds to $P_a' = 0.5$,

or

bias = Lower Specification Limit – X_T *(*at $P_a' = 0.5$*)*

The repeatability is determined by finding the differences of the reference value measurements corresponding $P_a' = 0.995$ and $P_a' = 0.005$ and dividing by an adjustment factor of 1.08[65].

$$Repeatability = \frac{X_T\ (at\ P_a^{'} = 0.995) \quad - \quad X_T\ (\ at\ P_a^{'} = 0.005)}{1.08}$$

To determine if the bias is significantly different from zero, the following statistic is used:

$$t = \frac{6.078 \times |Bias|}{\sigma_{repeatability}}$$

If this calculated value is greater than 2.093 ($t_{025,19}$), then the bias is significantly different from zero.

An example will clarify the attribute study data collection and calculation of repeatability and bias.

[65] The 1.08 adjustment (unbiasing) factor is specific for a sample size of 20 and was determined through a simulation of this approach with a 99% repeatability range. To convert the result to a 6 sigma range, divide by 5.15 and multiply by 6.

Example:

An attribute gage is being used to measure a dimension that has a tolerance of ± 0.010. The gage is an end-of-line 100% automatic inspection gage that is affected by repeatability and bias. To perform the attribute study, eight parts with reference values at intervals of 0.002 from –0.016 to –0.002 are run through the gage 20 times each. The number of accepts for each part are:

X_T	a
–0.016	0
–0.014	3
–0.012	8
–0.010	20
–0.008	20
–0.006	20
–0.004	20
-0.002	20

Since there are two reference values with $1 \le a \le 19$, at least four more parts must be found. Therefore, it is necessary to run parts with reference values at the midpoints of the existing intervals. These reference values and the number of accepts are:

–0.015	1
–0.013	5
-0.011	16

Now there are five reference values with $1 \le a \le 19$. The procedure requires that one more part be found with $1 \le a \le 19$. Therefore, the following part is evaluated:

–0.0105	18

Now that the data collection criteria have been satisfied, the probabilities of acceptance can be calculated using the binomial adjustments shown below.

X_T	a	P'_a
-0.016	0	0.025
-0.015	1	0.075
-0.014	3	0.175
-0.013	5	0.275
-0.012	8	0.425
-0.011	16	0.775
-0.0105	18	0.875
-0.010	20	0.975
-0.008	20	1.000

These probabilities are plotted on normal probability paper as shown in Figure III-C 4. By plotting a best-fit line through these points, the bias and repeatability can be determined. The bias is equal to the lower limit minus the reference value measurement that corresponds to $P_a' = 0.5$.

From Figure III-C 4:

$$bias = -0.010 - (-0.0123) = 0.0023$$

The repeatability is determined by finding the differences R of the reference value measurements corresponding to $P_a' = 0.995$ and $P_a' = 0.005$ and dividing by 1.08 From Figure III-C 4:

$$R = \frac{0.0084 \ (\ 0.0163 \ )}{1.08}$$

$$= \frac{0.0079}{1.08} = 0.0073$$

Then $\sigma_{repeatability} = \frac{R}{5.15}$ = 0.00142 and the associated *GRR* range is 0.0085

To determine if the bias is significantly different from zero, calculate:

$$t = \frac{6.078 \times |\text{Bias}|}{\sigma_{repeatability}}$$

$$= \frac{6.078 \times |0.0023|}{0.00142} = 9.84$$

Since $t_{0.025,19} = 2.093$, the bias is significantly different from zero.

Like the variable Gage Performance Curve shown in Chapter IV, Section F, the attribute *GPC* can also be plotted on plain graph paper (see Figure III-C 5). This can be accomplished in one of two ways. One approach would be to run the sample study at the other limit of the specification. In the example, the long method for attribute study would also have to be conducted at the high limit of the specification, and the calculated values plotted accordingly.

However, because of the previously mentioned assumptions, it would not be necessary to run the study again. Since the shape of the curve at the high limit should be a "mirror image" of the curve at the low limit, the only consideration necessary is the location of the curve with respect to X_T values. This location is determined by the bias.

The correct position of the curve would be defined at the point where the P_a' = 0.5, and the X_T value is equal to the specification limit minus the bias. In the example, this point would be $X_T = 0.010 - 0.0023 = 0.0077$. The *GPC* plotted in this manner is shown in Figure III-C 4.

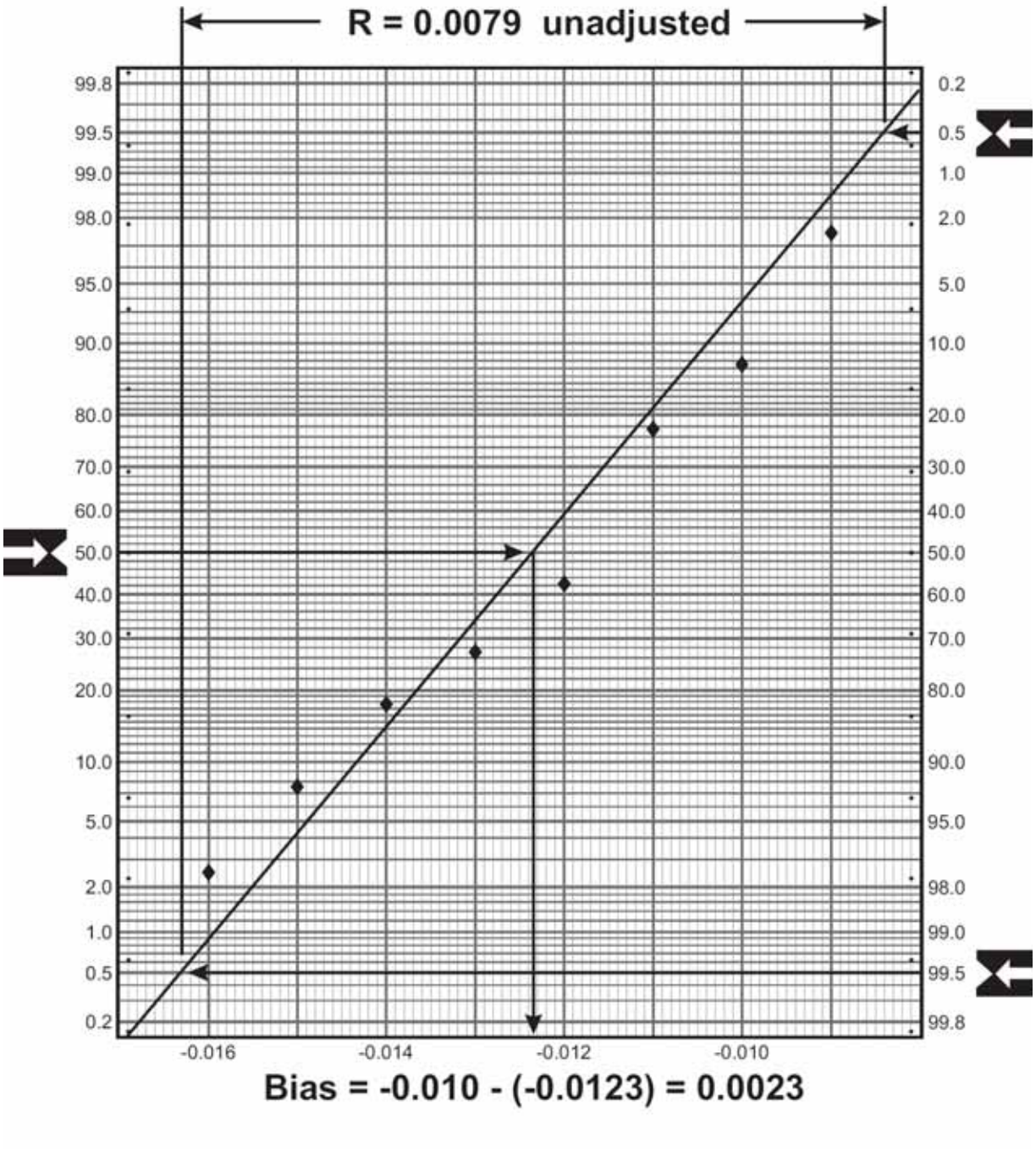


Figure III-C 4: Attribute Gage Performance Curve Plotted on Normal Probability Paper

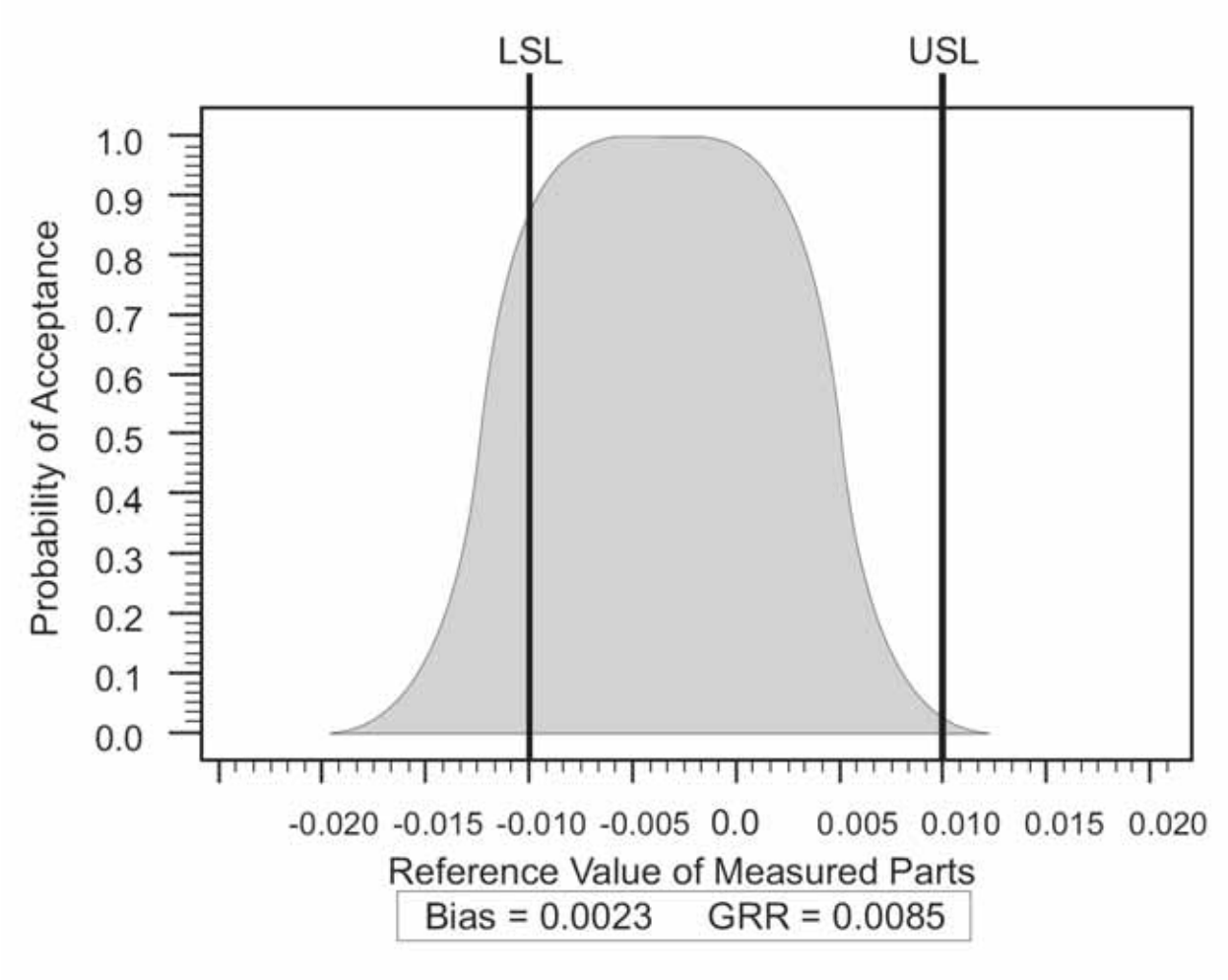


Figure III-C 5. Attribute Gage Performance Curve

CHAPTER IV

Other Measurement Concepts and Practices

Section A
Practices for Non-Replicable Measurement Systems

The focus of this reference manual is measurement systems where the readings can be replicated for each part. Not all measurement systems have this characteristic; for example:

- Destructive measurement systems
- Systems where the part changes on use/test.

> The following are examples of approaches to the analysis of measurement systems, including those not previously discussed in this manual. This is not intended to be a complete listing covering every type of measurement system but only examples of various approaches. If a measurement system does not fit the manual's focus, it is recommended that assistance from competent statistical resources be consulted.

Destructive measurement systems

When the part (characteristic) being measured is destroyed by the act of measuring the process is known as destructive measurement. This includes the whole class of measurement systems known as "destructive measurement systems"; for example destructive weld testing, destructive plating testing, salt spray/humidity booth testing, impact testing (gravelometer) or mass spectroscopy and other material characteristic testing processes.

These are the "classic" examples of a non-replicable measurement system since repeated readings cannot be taken on any single part.

Systems where the part changes on use/test

However, there are other measurement systems which are non-replicable where the part, itself, is not harmed by the measurement process but the characteristic being measure will change. Examples of this are: leak tests with qualitative data, testing using engine test stands, transmission test stands, vehicle dynamometers, etc.

Analysis of these systems will depend on whether

1) A homogeneous set of parts (small between part variation) can be found to represent a single part;
2) The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study – i.e., the measured characteristic does not change over the expected period of use; or
3) The dynamic (changing) properties can be stabilized

The mapping of the studies described in this chapter and the various scenarios are as follows:

Stability Studies					
Scenario	**S1**	**S2**	**S3**	**S4**	**S5**
The part is not changed by the measurement process; i.e., measurement systems that are non-destructive (replicable) and will be used with parts (specimens) with: • Static properties, or • Dynamic (changing) properties which have been stabilized.	✓	✓			
The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.	✓	✓			
Destructive measurement systems			✓	✓	
Other non-replicable measurement systems.			✓	✓	
Measurement systems with dynamic characteristics: e.g., test stands					✓

Variability Studies									
Scenario	**V1**	**V2**	**V3**	**V4**	**V5**	**V6**	**V7**	**V8**	**V9**
The part is not changed by the measurement process; i.e., measurement systems that are non-destructive and will be used with parts (specimens) with: • Static properties, or • Dynamic (changing) properties which have been stabilized.	✓								
Above with $p \geq 2$ instruments		✓							
Destructive measurement systems			✓	✓					
Other non-replicable measurement systems.			✓	✓					
Measurement systems with dynamic characteristics: e.g., test stands			✓	✓	✓	✓	✓	✓	
Measurement systems with dynamic characteristics: with $p \geq 3$ instruments									✓

Table IV-A 1: Methods Based on Type of Measurement System

Section B
Stability Studies

S1: Single Part[66], Single Measurement Per Cycle

Application:

a) Measurement systems in which the part is not changed by the measurement process; i.e., measurement systems that are non-destructive and will be used with parts (specimens) with:

- ✓ Static properties, or
- ✓ Dynamic (changing) properties which have been stabilized.

b) The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.

Assumptions:

- The measurement system is known (documented) to have a linear response over the expected range of the characteristic (property).
- Parts (specimens) cover the expected range of the process variation of the characteristic.

Analyze using *X* & *mR* charts:

- Determine measurement system stability:
  - ✓ Compare the plotted points to the control limits
  - ✓ Look for trends (*x* chart only)
- Compare $\sigma_e = \overline{R} / d_2^*$ (total measurement error) with the repeatability estimate σ_E from a variability study (see next section).
- Determine the bias if reference value is known:

$$bias = \bar{x} - reference\ value$$

[66] A reference standard can be used if it is appropriate for the process.

S2: $n \geq 3$ Parts[67], Single Measurement Per Cycle Per Part

Application:

a) Measurement systems in which the part is not changed by the measurement process; i.e., measurement systems that are non-destructive and will be used with parts (specimens) with:

- ✓ Static properties, or
- ✓ Dynamic (changing) properties which have been stabilized.

b) The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.

Assumptions:

- The measurement system is known (documented) to have a linear response over the expected range of the characteristic (property).
- Parts (specimens) cover the expected range of the process variation of the characteristic.

Analyze using a [*z, R*] chart: where $z_i = x_i - \mu_i$

and μ_i is the (reference) standard value or determined by the average of a large number of successive readings of the part (specimen).

- Determine measurement system stability:
  - ✓ Compare the plotted points to the control limits
  - ✓ Look for trends (*z* chart only)
- Compare[68] $\sigma_e = \overline{R} / d_2^*$ with the repeatability estimate σ_E from a variability study.
- Determine the bias if reference values are known:

$$bias = \overline{x} - reference\ value$$

- Determine the linearity if $n \geq 3$ parts were used:
  - ✓ The parts (specimens) must cover the expected range of the property
  - ✓ Each part (specimen) should be analyzed separately for bias and repeatability
  - ✓ Quantify the linearity using the linearity analysis discussed in Chapter 3, Section B

[67] A reference standard can be used if it is appropriate for the process.

[68] If more than one appraiser is involved in the data collection, then σ_E (repeatability estimate) is affected also by the reproducibility of the measurement system. Quantify reproducibility by scatter and whisker plots indexed by appraiser (see Chapter III, Section B).

If more than one instrument is use in this study, determine consistency (homogeneity of variation) among the instruments; e.g., use *F* test, Bartlett's test, Levene's test[69], etc.

S3: Large Sample from a Stable Process

Application:

The measurement system must be evaluating a homogeneous *independent identically distributed* ("*iid*") sample (collected and isolated). The measurements of individual parts (specimens) are not replicated so this study can be used with destructive and non-replicable measurement systems.

Assumptions:

- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., measured characteristic does not change over the expected period of use and/or storage.
- Parts (specimens) cover the expected range of the process variation of the characteristic (property).
- The measurement system's linearity is known (documented) over the expected range of the characteristic (property). (If the response is non-linear, the readings must be adjusted accordingly.)

Analyze by:

- Determining the total variability via a capability study with $n \geq 30$ parts. (This preliminary study should also be used to verify the consistency of the sample: i.e., all parts (specimens) come from a unimodal distribution.)
- $\sigma^2_{total} = \sigma^2_{process} + \sigma^2_{measurement\ system}$
- Measuring one or more individuals from the isolated sample per time period, use $\bar{x}$ & R or x & mR charts with the control limits determined by the capability study.
- Compare the plotted points to the control limits.
- Look for trends.
- Since the parts (specimens) do not change (an isolated sample), any indication of instability would be attributed to a change in the measurement system.

[69] *Dataplot*, National Institute of Standards and Technology, Statistical Engineering Division (www.itl.nist.gov).

S4: Split Specimens (General), Single Specimen Per Cycle

Application:

The measurements of individual parts (specimens) portions are not replicated so this study can be used with destructive and non-replicable measurement systems.

Assumptions:

- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., measured characteristic does not change over the expected period of use and/or storage.
- Parts (specimens) cover the extended range of the process variation of the characteristic (property).
- Specimens are split into *m* portions. With *m*=2 portions, this is often called a *test-retest study*.

Analyze using:

- Range chart to track the consistency of the measurements (confounded with the "within-lot" consistency).
- Compare $\sigma_e = \overline{R} / d_2^*$ with the repeatability estimate σ_E from a variability study.
- This is an upper bound study: $\sigma_e^2 = \sigma_E^2 + \sigma_{btwn}^2$
- Chart to track the consistency of the production process.

S4 with Pairs of Consecutive (Homogeneous) Parts from Different Lots – S4a

This study is the same as S4 with homogeneous parts from different lots. It is an upper bound study, since $\sigma_e^2 = \sigma_E^2 + \sigma_{btwn}^2 + \sigma_{lots}^2$

S5: Test Stands

In this situation, multiple measurement instruments (test stands) evaluate the same characteristics of a continual stream of product. The product stream is randomly assigned to individual stands.

S5a: Attribute Responses

Analyze using *p* charts:

- Determine consistency (of decisions) among the stands: a single chart including the results from all the stands.
- Determine stability within individual stands: separate chart for each stand.

Analyze the total system stability with a $\overline{p}$ & mR chart where $\overline{p}$ is the average over all the test stands in a given day.

S5b: Variable Data Responses

Analyze using ANOVA and graphical techniques:[70]

- Calculate $\overline{x}$ & s for each test stand (by characteristic), by time period.
- Determine consistency among the stands: a single $\overline{x}$ & s chart including the results from all the stands.
- Determine stability within individual stands: separate $\overline{x}$ & s chart for each stand.
- Quantify the consistency (homogeneity of variation) among the stands; e.g., use *F* test, Bartlett's test, Levene's test, etc.
- Determine if all stands are on the same target by comparing stand averages; e.g., using a one-way ANOVA analysis. If any difference exists, isolate different stands by using, for example, Tukey's *T* test.

[70] see also James, P. D., "Graphical Displays of Gage R&R Data," AQC Transaction, ASQC, 1991

Section C
Variability Studies

All descriptive studies are *enumerative* in nature in that they describe the measurement system (including the effects of the environment) during the study. Since measurement systems are to be used in making future decisions about products, processes, or services, an *analytic* conclusion about the measurement system is necessary. The transition from enumerative to analytic results requires subject matter knowledge and expertise to:

- Assure that all expected measurement sources of variation are considered in the design and execution of the study.
- Analyze the results (data) in light of the expected uses, environment, control, maintenance, etc.

V1: Standard *GRR* Studies

These studies are those contained within this reference manual. These studies include graphical analysis as well as numerical analysis.

V1a – Range Method (*R&R*)

V1b – Range Method (*R&R* and Within-Part)

V1c – ANOVA Method

V1d – Modified ANOVA/Range Method

V2: Multiple Readings With $p \geq 2$ Instruments

This allows the comparison of multiple instruments.

Application:

a) Measurement systems in which the part is not changed by the measurement process; i.e., measurement systems that are non-destructive and will be used with parts (specimens) with:

   1) Static properties, or

   2) Dynamic (changing) properties which have been stabilized.

Assumptions:

- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.
- Parts (specimens) cover the expected range of the process variation of the characteristic.

Analyze using Grubbs'[71] (or Thompson's)[72] estimates:

- Process variability
- Instrument variability = repeatability
- Confidence region calculations are available

V3: Split specimens (*m* = 2)

Application:

The measurements of individual parts (specimens) portions are not replicated so this study can be used with destructive and non-replicable measurement systems and can be used to analyze measurement systems with dynamic characteristics.

Assumptions:

- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., measured characteristic does not change over the expected period of use and/or storage.
- Parts (specimens) cover the extended range of the process variation of the characteristic (property).
- Specimens are split into *m* portions. With *m*=2 portions, this is often called a *test-retest study*.

Analyze using regression techniques:

- Estimate repeatability with the error term: $\sigma_E = \sigma_e$
- Linearity (by comparing estimated line with 45° line)

V3a – V3 with Pairs of Consecutive Parts

This study is the same as V3 using consecutive pairs of part rather than split specimens. This study is used in situations where the part cannot be split without destroying the property to be measured.

This is an upper bound study: $\sigma_E \leq \sigma_e + \sigma_{btwn}$

[71] See Reference List, Grubbs, F. E., 1973.
[72] See Reference List, Thompson, W. A., Jr., 1963.

V4: Split Specimens (General)

Application:

The measurements of individual parts (specimens) portions are not replicated so this study can be used with destructive and non-replicable measurement systems and can be used to analyze measurement systems with dynamic characteristics.

Assumptions:

- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., measured characteristic does not change over the expected period of use and/or storage.
- Parts (specimens) cover the extended range of the process variation of the characteristic (property).
- Split specimens into m portions where $m = 0 \bmod 2$ or 3; $m \geq 2$ (e.g., m =3, 4, 6, 9,.....).

Analyze using:

- Standard *GRR* techniques including graphics
- ANOVA – Randomized Block Design (two-way ANOVA)

V4a – V4 with Pairs of Consecutive (homogeneous) Parts from Different Lots

This study is the same as V4 using consecutive pairs of part rather than split specimens. This study is used in situations where the part cannot be split without destroying the property to be measured.

This is an upper bound study: $\sigma_E \leq \sigma_e + \sigma_{parts} + \sigma_{lots}$

> The Following Studies Assume the Part (specimen) Characteristic (property) is Dynamic.

V5: Same as V1 with Stabilized Parts

The parts used in the study are stabilized using a process based on engineering knowledge and expertise; e.g., engines which are 'broken-in' versus 'green' engines.

V6: Time Series Analysis

Assumptions:

- Repeated readings are taken over specified time intervals .
- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.
- Parts (specimens) cover the expected range of the process variation of the characteristic.

Analyze by determining the degradation model for each sample part:

- $\sigma_E = \sigma_e$
- Consistency of degradation (if $n \geq 2$)

V7: Linear Analysis

Assumptions:

- Repeated readings are taken over specified time intervals.
- The degradation in the measurement system is known (documented) to have a linear response over the specified time intervals.
- The shelf life of the characteristic (property) is known and extends beyond the expected duration of the study; i.e., the measured characteristic does not change over the expected period of use.
- Parts (specimens) cover the expected range of the process variation of the characteristic.

Analyze by linear regression:

- $\sigma_E = \sigma_e$
- Consistency of degradation (if $n \geq 2$)

V7a – V7 with a Homogeneous Sample

Analyze by linear regression:

This is an upper bound study: $\sigma_E \leq \sigma_e + \sigma_{btwn}$

V8: Time versus Characteristic (Property) Degradation

V6 & V7 can be modified to determine if the degradation is time (i.e., shelf life) or activity dependent.

V9: V2 with Simultaneous Multiple Readings and $p \geq 3$ Instruments

Analyze same as V2 (see also Lipson & Sheth, sec 13.2).

Section D
Recognizing the Effect of Excessive Within-Part Variation

Understanding the sources of variation of a measurement system is important for all measurement applications but becomes even more critical where there is significant within-part variation. Within-part variation, such as taper or out-of-round, can cause the measurement system evaluation to provide misleading results. This is because unaccounted within-part variation affects the estimate of repeatability, reproducibility, or both. That is, the within-part variation can appear as a significant component of the measurement system variation. Understanding the within-part variation present in the product will result in a more meaningful understanding of the suitability of the measurement system for the task at hand.

Examples of within-part variation which may be encountered are: roundness (circular runout), concentricity, taper, flatness, profile, cylindricity, etc.[73] It is possible that more than one of these characteristics may be present at the same time within the same part (composite error). The strength of each characteristic and their interdependencies may compound the data and the resultant understanding of the measurement system. Recognition and analysis of these additional sources of variation during a measurement system study is paramount to understanding the actual measurement system variation. A D.O.E., ANOVA or other more sophisticated statistical technique may be required to analyze this situation. Whatever methodology is chosen should be approved by the customer supplier quality representative.

Also, decisions that product design engineers make can unwittingly affect how a part is measured, how a fixture may be designed, and the result can affect the outcome of measurement error. An example might be a plastic part that has a critical feature on the parting line (a parting line typically has excess plastic material where the two halves of the mold join, and thus is an uncontrolled surface). These factors ought to be considered during a Design FMEA.

[73] Many of these features are controlled in the design by way of *Geometric Dimensioning and Tolerancing* (GD&T). GD&T provides an operationally defined method to check parts in a *functional* manner. Generally, a functional check is an attribute check. Where variable data is required, issues can arise as to using a gage designed for a functional check to yield variable data. This may sometimes be done by using the functional gage as a holding fixture for a CMM study. However, when this is done it is critical that the fixture hold the part firmly and repeatably in the same location (if it does not, the resultant MSA study should generate this error).

Once the within-part variation components are understood, it may be possible to control these factors within the measurement system (e.g., by redesigning the gage or using different fixturing methods/equipment) so that future data is not confounded.

Section E
Average and Range Method – Additional Treatment

There is an additional consideration worthy of mention relative to the Average and Range method of measurement system assessment.[74]

The primary purpose of this graphical approach is the same as other well designed measurement system analyses: to determine if the measurement process is adequate to measure the manufacturing process variation and/or evaluate product conformance

- Are all gages doing the same job?
- Are all appraisers doing the same job?
- Is the measurement system variation acceptable with respect to the process variation?
- How good are the data obtained from the measurement process or into how many non-overlapping groups or categories can the data be divided?

Procedural Steps

1) Care should be taken to follow the "Preparation for Measurement Systems Study", Chapter II, Section C.
2) Have each appraiser check each sample for the characteristic being studied. Record the first checks on the top data row of a control chart (see Figure IV-E 1 & 2).
3) Repeat the checks and record the data on the second data row of the control chart. *(Note: Do not allow the appraisers to see their original reading while making this second check.)* The data should now show two checks on the same part by each appraiser.
4) Analyze the data by calculating the average ($\bar{X}$) and range (R) for each subgroup.
5) Plot the range values on the range chart and calculate the average range ($\bar{R}$) (include all sub-group ranges (R) for all appraisers). Draw this average range on the chart. Use the D_4 factor for n = 2 to calculate the control limit for the range chart. Draw in this limit and determine if all values are in control.
    - ✓ If all ranges are in control, all appraisers are doing the same job.If one appraiser is out of control, his method differs from the others.
    - ✓ If all appraisers have some out-of-control ranges, the measurement system is sensitive to appraiser technique and needs improvement to obtain useful data.

[74] The control chart example is taken with permission from "Evaluating the Measurement Process," by Wheeler & Lyday (see Reference List).

6) Next, plot the average for each subgroup ($\bar{X}$) for all appraisers on the average chart (see Figure IV-E 1 & 2). The average values represent both variation and measurement variation.

Calculate the grand average ($\bar{\bar{X}}$) (include all subgroup averages ($\bar{X}$) for all appraisers). Draw this grand average ($\bar{\bar{X}}$) on the chart.

Now calculate the control limits for this chart using the A_2 factor for n = 2 and average range ($\bar{R}$) from the range chart and draw these limits on the average chart. Note in this study, the range chart contains only measurement variation. Thus, the area between the control limits of the averages represents the amount of measurement variation in the system.

If all averages fall inside the control limits, the measurement variation overshadows the process variation. In other words, the measurement process has more variation than the manufacturing process and is of **no** value in monitoring or controlling that process.

If less than half of the averages are outside the limits, the measurement system is inadequate for process control.

On the other hand, if a majority of the averages fall outside the control limits, it indicates that the signals from the manufacturing process are greater than measurement variation. This measurement system can provide useful data for controlling the process.

Worksheet Example

The question, "How good are the data collected by this measurement system?" can be answered by completing the worksheet example, Figure IV-E 3 & 4. All data needed for the worksheet can be found on the average and range charts described above.

Following are the steps used in completing the worksheet example (Figure IV-E 3 & 4):

1) Identify the measurement and characteristic being evaluated, who is doing the study and the date of the study.
2) The average subgroup range ($\bar{R}$) is obtained directly from the control chart.
3) The number of replications (r) is the number of times each appraiser checked the **same** part.
4) Calculate the estimated repeatability standard deviation ($\hat{\sigma}_e$), as shown in Figure IV-E 3 by using the d_2 value for the corresponding r.
5) Insert the number of appraisers (n_A) in the space provided.
6) Insert the number of samples (n) in the space provided.

7) Compute each appraiser average by averaging all the samples obtained by **each** appraiser and enter these averages in the space provided for each appraiser (*A, B, C*).

8) Examine the appraiser averages (*A, B, C*) and determine the range of the appraiser averages by subtracting the lowest from the highest and insert in the space for (R_A).

9) Compute the estimated appraiser standard deviation ($\hat{\sigma}_A$) as shown by using the d_2^* value for the corresponding n_A value.

10) Compute the sample averages by averaging the value obtained by all appraisers for **each** sample. For example, *(sample 1 avg. of appraiser A + sample 1 avg. of appraiser B + sample 1 avg. of the last appraiser* and divide this sum by the *number of appraisers*). This is the best estimate of that sample's true value. Place the value for each sample in the space provided (1, 2, 3.....9, 10) in Figure IV-E 4.

11) Observe the sample averages (1, 2, 3.....9, 10) and calculate the range of the sample averages (R_P) by subtracting the lowest from the highest. Insert this value in the space provided.

12) Estimate the sample-to-sample standard deviation ($\hat{\sigma}_P$) as shown by using the d_2^* value for the corresponding *n* value.

13) Compute the *"Signal-to-Noise Ratio"* (*SN*) by dividing the sample standard deviation by the measurement standard deviation and insert in the space provided.

$$SN = \frac{\sigma_P}{\sigma_{GRR}}$$

14) Determine the number of distinct product categories that can be distinguished by these measurements. Multiply *SN* by 1.41 and insert in the space provided (Figure IV-E 4).

> Only the integer portion of this number need be considered since it is defining distinct categories. (Drop all decimal fractions.) (See Figure IV-E 4.)

If the number of categories is less than two (2), the measurement system is of no value in controlling the process. It is all noise and one part cannot be said to be different from another.

If the number of categories is two (2), it means the data can be divided into only high and low groups; however this is only the equivalent of attribute data.

If the number of categories is three (3), the data can be divided into high, medium and low groups. This is a slightly better measurement system.

A system containing four (4) or more categories would be much better than the first three examples.

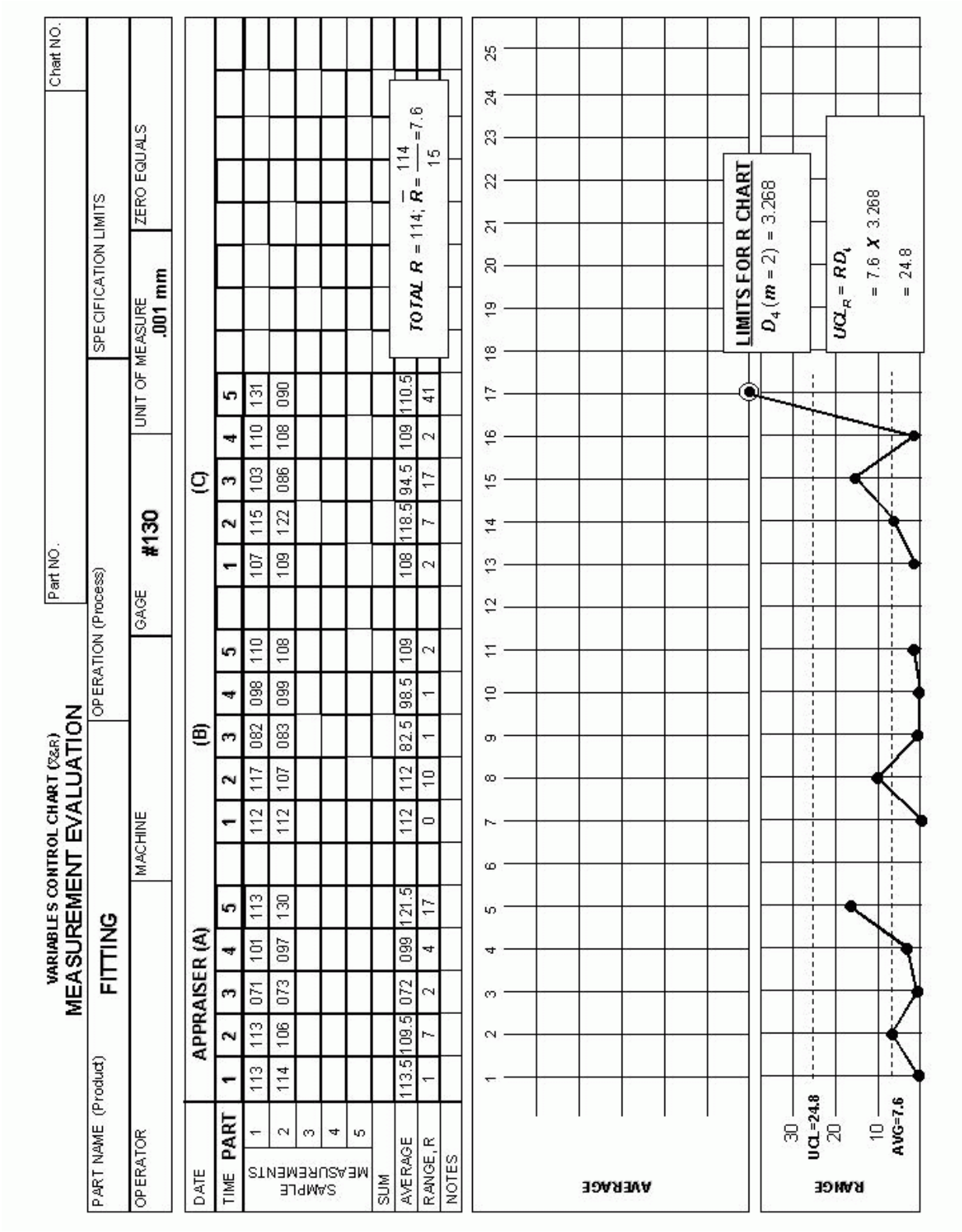


Figure IV-E 1: Measurement Evaluation Control Chart ($\bar{X}$ & R) - 1

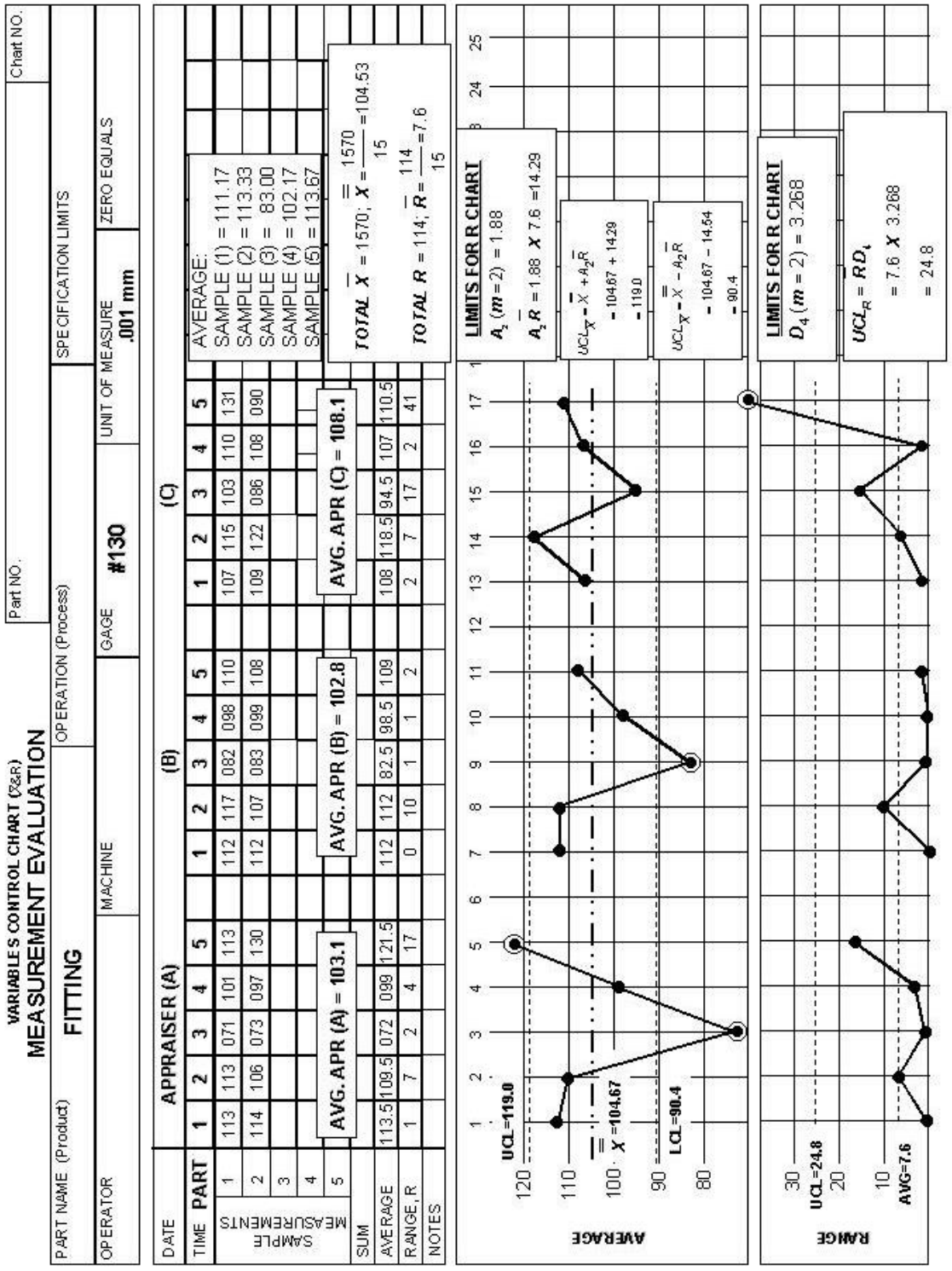


Figure IV-E 2: Measurement Evaluation Control Chart ($\bar{X}$ & R) - 2

Measurement Evaluated_____**FITTING LENGTH**____ **Done By**_____**R.W.L.**_____ **Date**__**MM-DD-YY**____

REPLICATION ERROR: Average Subgroup Range $= \bar{R} = 7.6$

Number of Replications = Subgroup Size = $r = 2$

Estimate Replication Standard Deviation $\frac{\bar{R}}{d_2} = \hat{\sigma}_e = 6.738$

r	d_2
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078

CALCULATIONS FOR APPRAISER EFFECT:

Write appraiser averages below:

n_A	d_2^*
2	1.410
3	1.906
4	2.237
5	2.477
6	2.669
7	2.827
8	2.961
9	3.076
10	3.178

Number of Appraisers $= n_A = 3$

Number of Samples $= n = 5$

$108.1 - 102.8 = R_A$

Range of Appraiser Averages $= R_A = 5.3$

$\frac{5.3}{1.906} = \hat{\sigma}_A$ $\quad \frac{\bar{R}}{d_2^*} = \hat{\sigma}_A = 2.781$

Appraiser	Average
A	103.1
B – lo	102.8
C – hi	108.1
D	____
E	____
F	____

Figure IV-E 3: Alternate Computations for Evaluating a Measurement Process (Part 1 of 2).

CALCULATIONS FOR MEASUREMENT ERROR STANDARD DEVIATION:
Estimate the measurement error standard deviation to be:

$$\hat{\sigma}_m = \sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_A^2}$$
$$= \sqrt{(6.7376)^2 + (2.7806)^2}$$
$$= 7.289$$

Write the appropriate value for $\hat{\sigma}_m$ here: $\boxed{\hat{\sigma}_m = 7.289}$

CALCULATIONS FOR SIGNAL TO NOISE RATIO:

n	d_2^*
2	1.410
3	1.906
4	2.237
5	2.477
6	2.669
7	2.827
8	2.961
9	3.076
10	3.178

Write the average for each sample piece or batch below:

Sample	Average
1	111.17
2	113.33
3 - Lo	83.00
4	102.17
5 - Hi	113.67
6	_____
7	_____
8	_____
9	_____
10	_____

$113.67 - 83.0 = \boldsymbol{R_p}$

Range for these Sample Averages $= \boldsymbol{R_p} = 30.67$

Estimate Sample to Sample Standard Deviation:

$\frac{30.67}{2.477} = 12.38$ $\quad \frac{\boldsymbol{R_P}}{\boldsymbol{d_2^*}} = \boxed{\hat{\sigma}_P = 12.38}$

Signal to Noise Ratio:

$\frac{12.38}{7.289} = 1.698$ $\quad \frac{\hat{\sigma}_P}{\hat{\sigma}_m} = 1.698$

Thus the number of distinct product categories that can be Reliably distinguished by these measurements is 1.41 x 1.698

$$1.41 \left(\frac{\hat{\sigma}_P}{\hat{\sigma}_m} \right) = 2.395 \quad \textbf{or} \quad 2$$

This is the number of non-overlapping 97% confidence intervals that will span the range of product variation. (A 97% confidence interval centered on a single measurement would contain the actual product value that is represented by that measurement 97% of the time.)

Figure IV-E 4: Alternate Computations for Evaluating a Measurement Process (Part 2 of 2).

Section F
Gage Performance Curve[75]

The purpose of developing a Gage Performance Curve (*GPC*) is to determine the probability of either accepting or rejecting a part of some reference value. Ideally, the *GPC* for a measurement without error is shown in Figure IV-F 1. However, this is the ideal for measurement systems, rather than what normally occurs.

Once the amount of error has been determined, it is possible to calculate the probability of accepting a part of some reference value when using that system.

To accomplish this, the assumption is made that the measurement system error consists primarily of lack of repeatability, reproducibility and bias. Further, repeatability and reproducibility are taken to be normally distributed with some variance, σ^2. Consequently, gage error is normally distributed with a mean X_T, the reference value, plus the bias, and has some variance, σ^2. In other words:

$$Actual\ Value\ \ from\ the\ Gage = N(X_T + b, \sigma^2)$$

The probability of accepting a part of some reference value is given by the relationship

$$P_a \ = \ \int_{LL}^{UL} N\ (X_T\ +\ b,\ \sigma^2)dx$$

Using the standard normal table

$$P_a \ = \ \phi\left(\frac{UL\ -\ (X_T\ +\ b)}{\sigma}\right)\ -\ \phi\left(\frac{LL\ -\ (X_T\ +\ b)}{\sigma}\right)$$

where

$$\phi\left(\frac{UL\ -\ (X_T\ +\ b)}{\sigma}\right) = \int_{-\infty}^{UL} N\ (X_T\ +\ b,\ \sigma^2\ )dx$$

$$\phi\left(\frac{LL\ -\ (X_T\ +\ b)}{\sigma}\right) = \int_{LL}^{\infty} N\ (X_T\ +\ b,\ \sigma^2\ )dx$$

[75] Adapted with permission from "Analysis of Attribute Gage Systems" by J. McCaslin & G. Gruska, ASQC, 1976.

Example:

Determine the probability of accepting a part when the reference torque value is 0.5 Nm, 0.7 Nm, 0.9 Nm.

Using data from a previously conducted study:

Upper specification = USL = 1.0 Nm

Lower specification = LSL = 0.6 Nm

bias = b = 0.05 Nm

σ_{GRR} = 0.05 Nm

Applying the above to the formulas on the previous page:

$$P_a = \phi\left(\frac{UL - (X_T + b)}{\sigma}\right) - \phi\left(\frac{LL - (X_T + b)}{\sigma}\right)$$

$$P_a = \phi\left(\frac{1.0 - (0.5 + 0.05)}{0.05}\right) - \phi\left(\frac{0.6 - (0.5 + 0.05)}{0.05}\right)$$

$$P_a = \phi(9.0) - \phi(1.0)$$

$$= 1.0 - 0.84$$

$$= 0.16$$

That is, when the part has a reference value of 0.5 Nm it will be rejected approximately 84% of the time.

Gage Performance Curve Example:

For X_T = 0.7 Nm

$$P_a = \phi\left(\frac{1.0 - (0.7 + 0.05)}{0.05}\right) - \phi\left(\frac{0.6 - (0.7 + 0.05)}{0.05}\right)$$

$$P_a = \phi(5.0) - \phi(-3.0)$$

$$= 0.999$$

If the reference value of the part is 0.7 Nm then it will be rejected less than approximately 0.1% of the time.

For X_T = 0.9 Nm

$$P_a = \phi\left(\frac{1.0 - (0.9 + 0.05)}{0.05}\right) - \phi\left(\frac{0.6 - (0.9 + 0.05)}{0.05}\right)$$

$$P_a = \phi(1.0) - \phi(-7.0)$$

$$= 0.84$$

If the reference value of the part is 0.9 Nm then it will be rejected approximately less than 16% of the time.

If the probability of acceptance is calculated for all values of X_T and plotted, then the Gage Performance Curve, as shown in Figure IV-F 2, would be obtained.

This same curve can be more easily plotted on normal probability paper, as shown in Figure IV-F 3. Once plotted, the *GPC* gives the probability of accepting a part of any part size.

In addition, once *GPC* is developed, it can be used to calculate the repeatability and reproducibility error, and the bias error[76].

The 6 *GRR* range can be determined by finding the X_T value that corresponds to $P_a = 0.99865$ (z = 3), and the X_T value that corresponds to $P_a = 0.00135$ (z = -3) for either of the two limits. The *GRR* is the difference between the two X_T values, as shown graphically in Figure IV-F 3.

An estimate of the bias is determined by finding the X_T, for either the upper or lower limit, that corresponds to $P_a = 0.5$, and calculating:

$$B = X_T - LSL \quad \text{or} \quad BB = X_T - USL$$

depending on which limit X_T is chosen.[77]

[76] See "Attribute Gage Study," Chapter III, Section C.
[77] This assumes that the measurement system is linear over the operating range.

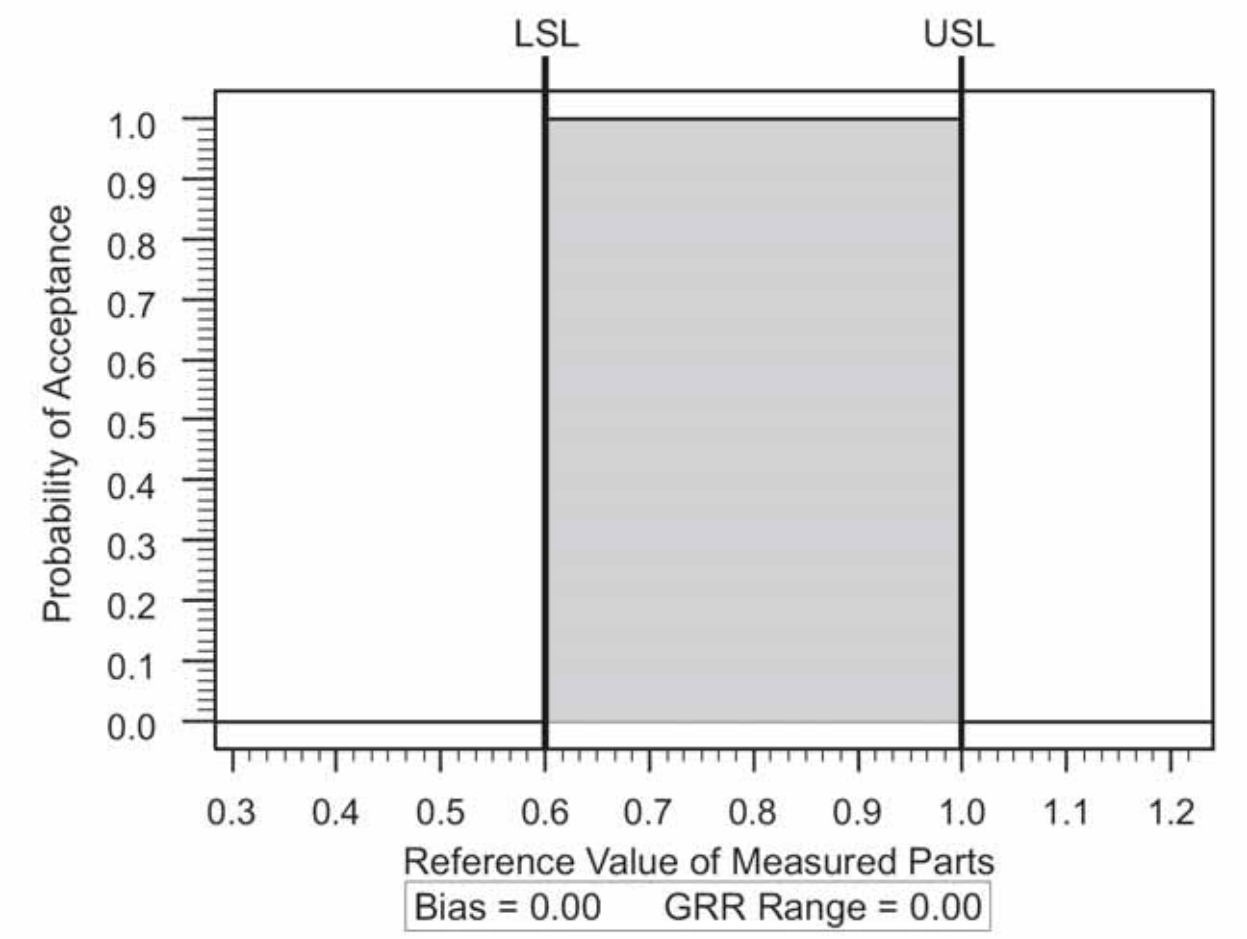


Figure IV-F 1: Gage Performance Curve Without Error

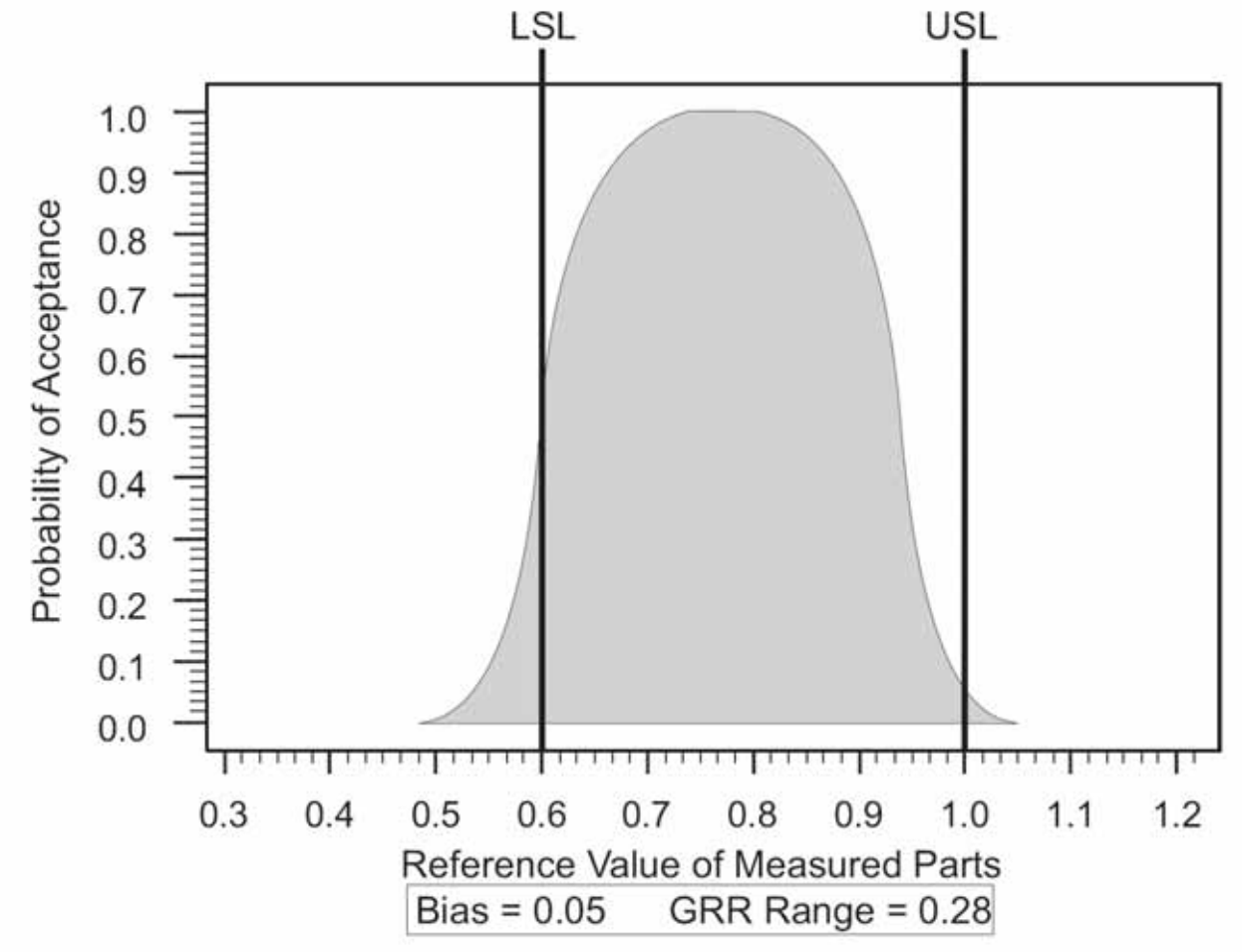


Figure IV-F 2: Gage Performance Curve –Example

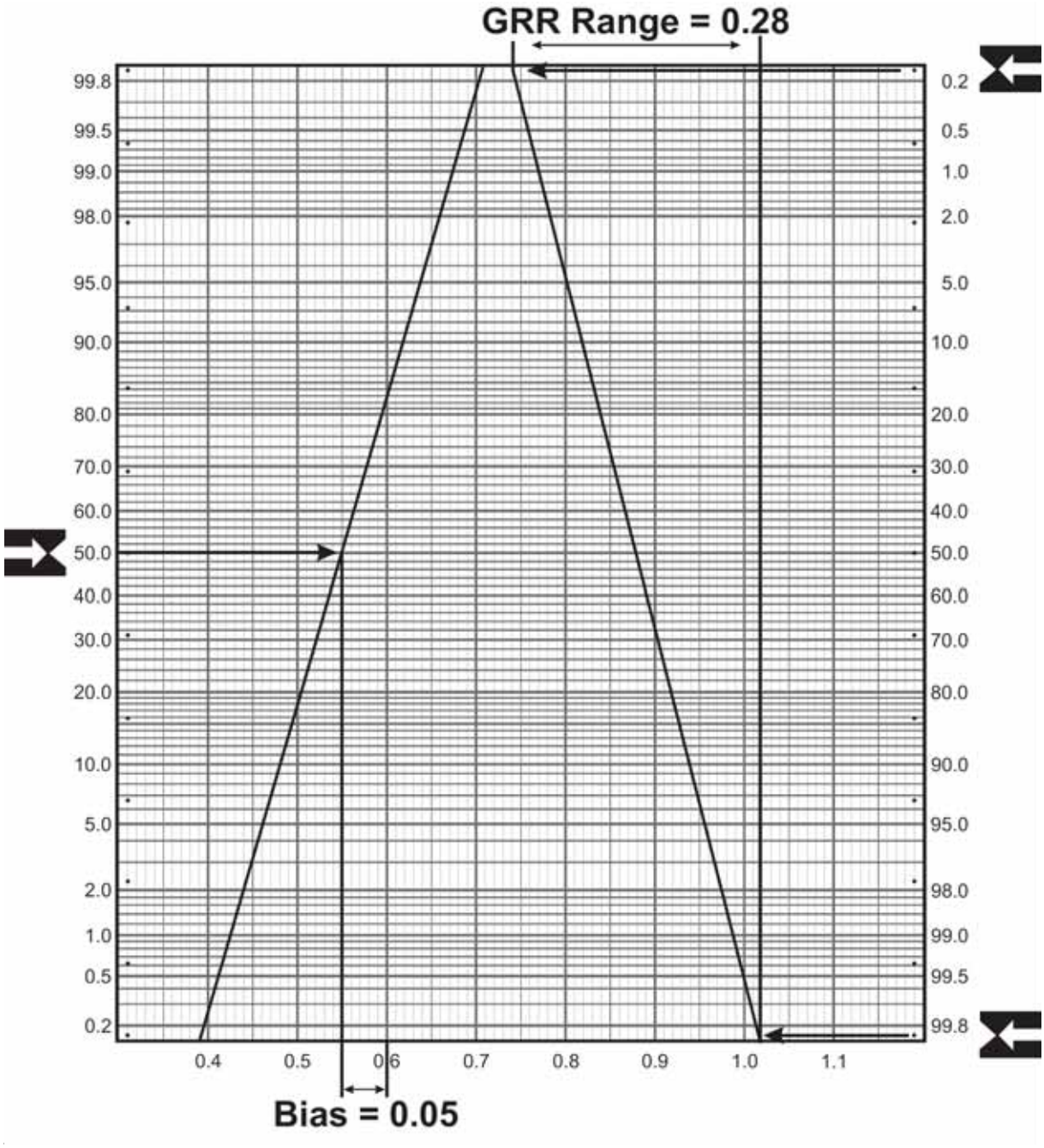


Figure IV-F 3: Gage Performance Curve Plotted on Normal Probability Paper

Section G
Reducing Variation Through Multiple Readings

If the variation of the present measurement system is not acceptable (over 30%), there is a method that can be used to reduce the variation to an acceptable level until proper measurement system improvements can be made. The unacceptable variation may be reduced by taking multiple statistically independent (non-correlated) measurement readings of the part characteristic being evaluated, determining the average of these measurements and letting the numeric value of the result be substituted for the individual measurement. This method will, of course, be time consuming, but is an alternative (with customer approval), until improvements are made to the measurement system (i.e., redesigning or buying a new system). The procedure for this alternative method is as follows:

1) Determine the number of multiple readings required to meet an acceptable level of variation.
2) Follow the gage study procedure discussed earlier in this guideline.

Example:

In the XYZ Corporate example, the percentage of tolerance *GRR* variation is 25.5%, with a 6 sigma spread equal to 0.24. The customer wants to reduce this figure to at least 15%, which would have an equivalent 6 sigma spread of 0.14.[78]

To determine the number of multiple readings required to meet the desired 15% criteria, it must first be understood that the distribution of individual and average measurements has the same mean numeric value. Secondly, the variance of the distribution of averages is equal to the variance of the distribution of individuals divided by the sample size. Understanding this relationship as shown below, the number of multiple readings required can then be determined.

$$(6\sigma_{\bar{x}})^2 = \frac{(6\sigma)^2}{n}$$

This can be approximated by

$$(6s_{\bar{x}}) = \frac{(6s)}{\sqrt{n}}$$

and

$$0.14 = \frac{0.24}{\sqrt{n}}$$

[78] See note on page vi.

so

$$\sqrt{n} = 1.714$$

and

$$n = 3 \text{ (rounded to nearest integer)}$$

Therefore, 3 multiple readings on the part characteristic will reduce the total measurement system variation to approximately 0.14 and the *%GRR* to 15%.

This method should be considered as a temporary step until other improvements are made on the measurement system. This should be used only with the concurrence of the customer.

Section H
Pooled Standard Deviation Approach to *GRR*[79]

Analysis of a measurement system usually assumes that replicated data from all the part/specimens can be obtained from all the appraisers in a random fashion. This may not always be possible. If the measurement system analysis includes multiple locations it can be logistically unfeasible to demand a random sampling. Also some tests, notably chemical and metallurgical analyses (inter- and intra- laboratory studies), may require a cross section of diverse samples which are not part of a homogeneous process and may not be available at the same time.

These situations may be handled by using a nested DOE. An alternate approach is the pooled standard deviation *GRR* study which follows the methodology discussed in ASTM E691.

This approach views each part as a separate material and then computes the repeatability and reproducibility standard deviations as in E691. This will yield multiple separate values of repeatability and reproducibility. Since the parts are considered to be essentially identical, these separate estimates are assumed to be effectively identical. Of course they will never be exactly the same, but their average will give a good estimate of the true level of repeatability and similar reproducibility.

> If this approach is used to evaluate a group of laboratories there is a concern of just what the nature of the "reproducibility" is. If this is greater than zero most of the time (that is for most of the materials) it should be interpreted as implying that there are differences between operators; i.e., in an interlaboratory program this would suggest that there are real differences between labs.

Sequential Application

Although the E691 approach is typically used with a complete sample, it lends itself to a sequential approach. This is useful when all the samples are not available at the same time. It also can be used as part of the calibration process to maintain information on the measurement system's variability.

The following study description assumes that the study will be applied in a sequential manner.

Conducting the Study

Care should be taken to follow the "Preparation for Measurement System Study" shown in Chapter II, Section C.

Continue from step 6 on page 74:

[79] Portions of this section including all of "Consistency Statistics" contributed by Neil Ullman of the American Society for Testing and Materials (ASTM *International*).

7) Have each of the $m \geq 2$ appraisers evaluate the part for $r \geq 3$ readings. Record the data on the appropriate rows of a data collection sheet (see Sample Forms). *(Note: Do not allow the appraisers to see their original reading while making these multiple checks.)*

8) Calculate the average ($\bar{X}$) and standard deviation (*s*) for the new part for each appraiser.

9) Plot the standard deviation values on the standard deviation chart and calculate the average standard deviation ($\bar{s}$) (include all sub-group standard deviation for all appraisers). Draw this average standard deviation on the chart. Use the B_4 factor for *r* samples to calculate the upper control limit for the standard deviation chart. Draw in this limit and determine if all values are in control (see Figure IV-H 1).

10) Plot the average ($\bar{X}$) for each subgroup for all appraisers on the average chart (see Figure IV-H 1). The average values represent both process variation and measurement variation.

11) Calculate the grand average ($\bar{\bar{X}}$) (include all subgroup averages ($\bar{X}$) for all appraisers). Draw this grand average ($\bar{\bar{X}}$) line on the chart.

12) Calculate the control limits for this chart using the A_2 factor for *r* and average standard deviation ($\bar{s}$) from the standard deviation chart; draw these limits on the average chart.

13) Analyze the data using the control charts and other graphical techniques as discussed in the Average and Range Method (see Chapter III).

14) Evaluate the measurement system's parameters for each part by pooling the appraisers' results.

$$s_{\bar{\bar{X}}_g} = \sqrt{\frac{\sum_{i=1}^{m} s_{\bar{X}_i}^2}{m}}$$

$$Repeatability_g = s_{E_g} = \sqrt{\frac{\sum_{i=1}^{m} s_{s_i}^2}{m}}$$

$$Reproducibility_g = s_{A_g} = \sqrt{s_{\bar{X}_g}^2 - \frac{s_{E_g}^2}{m}}$$

$$GRR_g = s_{GRR_g} = \sqrt{s_{E_g}^2 + s_{A_g}^2}$$

E691 follows the convention wherein the MSA's reproducibility is referred to as the appraiser variation and the MSA's *GRR* is called reproducibility. In this case,

$$s_{appr} = \sqrt{s_{\bar{x}}^2 - \frac{s_r^2}{3}}$$

$$s_R = \sqrt{s_r^2 + s_{appr}^2}$$

where $s_r = s_E$ = Repeatability and $s_R = GRR$ = ASTM Reproducibility

15) Evaluate the overall measurement system's parameters by pooling the part results where g = number of parts.

$$Repeatability = s_E = \sqrt{\frac{\sum_{i=1}^{g} s_{E_i}^2}{g}}$$

$$Reproducibility = s_A = \sqrt{\frac{\sum_{i=1}^{g} s_{A_i}^2}{g}}$$

$$GRR = s_{GRR} = \sqrt{\frac{\sum_{i=1}^{g} s_{GRR_i}^2}{g}}$$

When calculating the percent of total variation, the historic standard deviation for the process should be used.

If the parts cover a wide variety of processes, for example different metallurgical or chemical specimens, the evaluation of percent total variation should be based on the process variation of specific specimens and not the total variation over all specimens.

Care should be given in the interpretation of the measurement system's parameters in the case where appraisers are located in different facilities (e.g., laboratories).

The Repeatability will include between-equipment variation as well as within-equipment variation. This can be evaluated by calculating and comparing the repeatability within each location.

The Reproducibility will include between-location variation as well as between-appraiser variation. These components cannot be separated with this study.

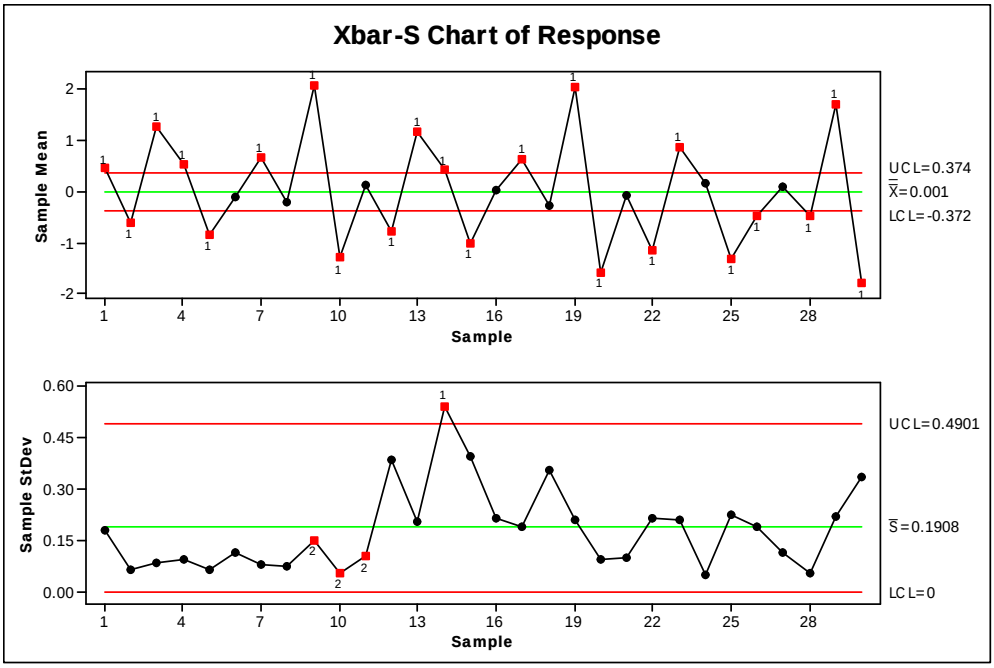


Figure IV-H 1: Pooled Standard Deviation Study Graphical Analysis[80]

[80] Control limits based on pooled standard deviation adjusted by an unbiasing constant

						Part									Across Parts
	Trial		1	2	3	4	5	6	7	8	9	10			
A	1		0.29	-0.56	1.34	0.47	-0.8	0.02	0.59	-0.31	2.26	-1.36			
	2		0.41	-0.68	1.17	0.5	-0.92	-0.11	0.75	-0.2	1.99	-1.25			
	3		0.64	-0.58	1.27	0.64	-0.84	-0.21	0.66	-0.17	2.01	-1.31			
		xbar	0.4467	-0.6067	1.2600	0.5367	-0.8533	-0.1000	0.6667	-0.2267	2.0867	-1.3067	xdbbar	0.19033333	
		sd	0.1779	0.0643	0.0854	0.0907	0.0611	0.1153	0.0802	0.0737	0.1504	0.0551	appr sd	0.10289153	
B	1		0.08	-0.47	1.19	0.01	-0.56	-0.2	0.47	-0.63	1.8	-1.68			
	2		0.25	-1.22	0.94	1.03	-1.2	0.22	0.55	0.08	2.12	-1.62			
	3		0.07	-0.68	1.34	0.2	-1.28	0.06	0.83	-0.34	2.19	-1.5			
		xbar	0.1333	-0.7900	1.1567	0.4133	-1.0133	0.0267	0.6167	-0.2967	2.0367	-1.6000	xdbbar	0.06833333	
		sd	0.1012	0.3869	0.2021	0.5424	0.3946	0.2120	0.1890	0.3570	0.2079	0.0917	appr sd	0.3017394	
C	1		0.04	-1.38	0.88	0.14	-1.46	-0.29	0.02	-0.46	1.77	-1.49			
	2		-0.11	-1.13	1.09	0.2	-1.07	-0.67	0.01	-0.56	1.45	-1.77			
	3		-0.15	-0.96	0.67	0.11	-1.45	-0.49	0.21	-0.49	1.87	-2.16			
		xbar	-0.0733	-1.1567	0.8800	0.1500	-1.3267	-0.4833	0.0800	-0.5033	1.6967	-1.8067	xdbbar	-0.25433333	
		sd	0.1002	0.2113	0.2100	0.0458	0.2223	0.1901	0.1127	0.0513	0.2194	0.3365	appr sd	0.19056058	
By Part results		sd xbar	0.26182	0.28005	0.19648	0.19751	0.24077	0.26555	0.32524	0.14385	0.21221	0.25125	**Pooled sd's**		
													Measurement System		
	Repeatability		0.13153	0.25721	0.17534	0.31863	0.26388	0.17736	0.13524	0.21252	0.19494	0.20385			
		pooled		0.204274	0.195107	0.23223	0.238896	0.229787	0.218795	0.218021	0.215578			**0.21443466**	
	Reproducibility		0.25056	0.23743	0.16839	0.07191	0.18644	0.24501	0.31573	0.07508	0.17991	0.22198			
		pooled		0.244086	0.221744	0.195372	0.193619	0.203089	0.222696	0.209998	0.206871			**0.20843064**	
	GRR		0.28299	0.35004	0.24310	0.32664	0.32310	0.30247	0.34347	0.22540	0.26527	0.30138			
		pooled		0.318285	0.295359	0.303481	0.307505	0.306671	0.312194	0.302708	0.29878			**0.29904106**	

Table IV-H 1: Pooled Standard Deviation Analysis Data Set

Consistency Statistics

The ASTM and ISO methods[81] suggest that two "consistency" statistics, *h* and *k*, be computed. The *h* values are calculated as:

$$h = \frac{\bar{x}_{appr} - \bar{\bar{x}}_{part}}{s_{\bar{x}}}$$

For appraiser A and part 1 the average ($\bar{x}_{appr}$ above) is 0.447 and the part average ($\bar{\bar{x}}_{part}$ above) is 0.169. The standard deviation among appraisers ($s_{\bar{x}}$ above) is 0.262. Then

$$h = \frac{0.447 - 0.169}{0.262} = \frac{0.278}{0.262} = 1.06$$

The value of *k* is the ratio of the standard deviation for each part for each appraiser to the repeatability standard deviation. In this case (appraiser A and part 1) it is:

$$k = \frac{\text{standard deviation (appr A, part 1)}}{\text{repeatability}} = \frac{0.178}{0.132} = 1.35$$

One reason these are computed is to allow comparisons among very different materials.

Although in this example there is not a collection of widely different materials which have different levels and possibly very different standard deviations, the *h* and *k* calculations of E691 can still use to compare the repeatability standard deviation and the response values by appraisers. In the following table the *h* and *k*'s are listed by appraiser.

h

	Part											
Appr	1	2	3	4	5	6	7	8	9	10	avg *h*	"*z*"
A	1.06	0.87	0.82	0.86	0.88	0.32	0.65	0.80	0.69	1.05	0.80	2.53
B	-0.14	0.22	0.29	0.24	0.21	0.80	0.50	0.32	0.46	-0.11	0.28	0.88
C	-0.93	-1.09	-1.11	-1.10	-1.09	-1.12	-1.15	-1.12	-1.15	-0.94	-1.08	-3.41

k

											median *k*	"*z*"
A	1.35	0.25	0.49	0.28	0.23	0.65	0.59	0.35	0.77	0.27	0.42	-3.20
B	0.77	1.50	1.15	1.70	1.50	1.20	1.40	1.68	1.07	0.45	1.30	3.14
C	0.76	0.82	1.20	0.14	0.84	1.07	0.83	0.24	1.13	1.65	0.84	-0.17

[81] See ISO 5725

In the last two columns are the averages and a value of "z-value" to see if the appraisers are significantly different. The *h* values indicate that appraiser A is significantly high and appraiser C is significantly low in their readings of the size of the parts. It is also this significant difference which creates the *GRR* standard deviation.

The repeatability standard deviations can be also evaluate by looking at the *k* values. To do this, compute the median *k* and then an approximate "z score". With this study, the expected median is about 0.861 with a standard deviation of approximately 0.439. The median *k* for appraiser A is then about –3.2 standard deviations below the expected level and appraiser B is as significantly high. So we see very great differences in the performance of just these three operators.

The graphs of *h* (Figure IV-H 2) and *k* (Figure IV-H 3) also help to illustrate these differences. Appraiser C has much lower results than the others. Similarly, the *k* values show how much lower appraiser A variation is with respect to repeatability. These are certainly issues to be examined about the performance of the measurement method as carried out by these appraisers.

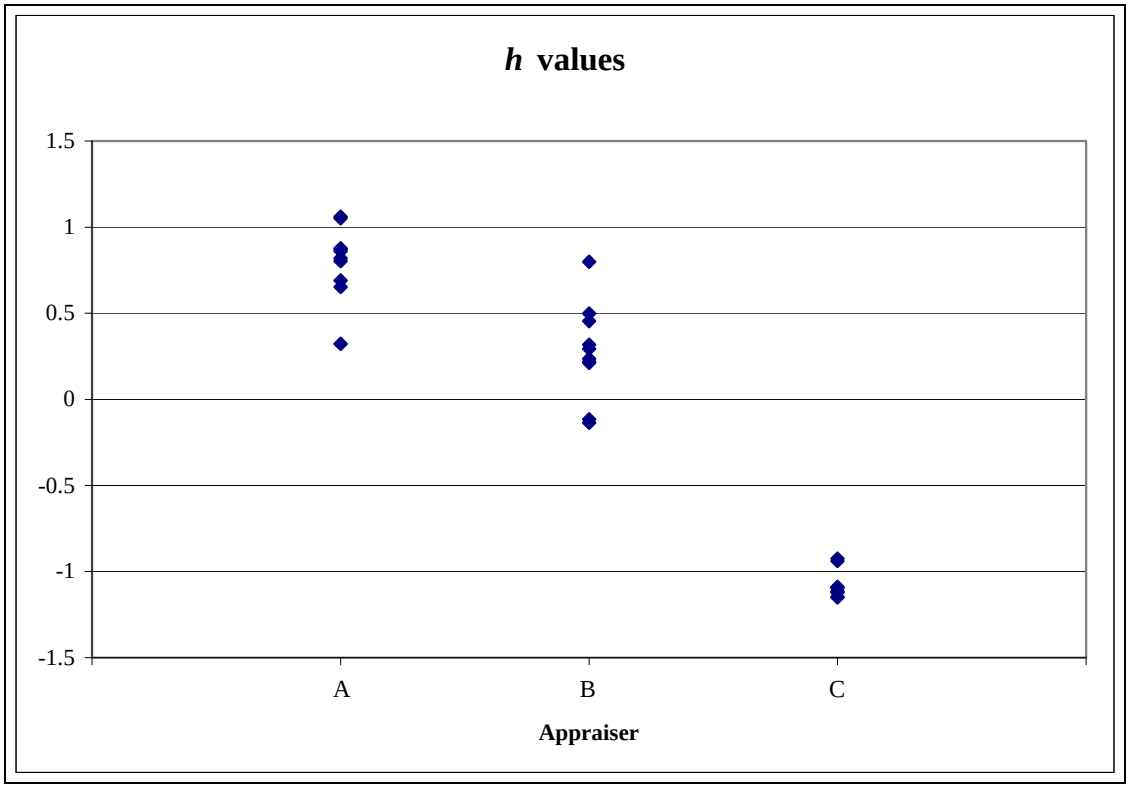


Figure IV-H 2: Dot diagram of *h* values.

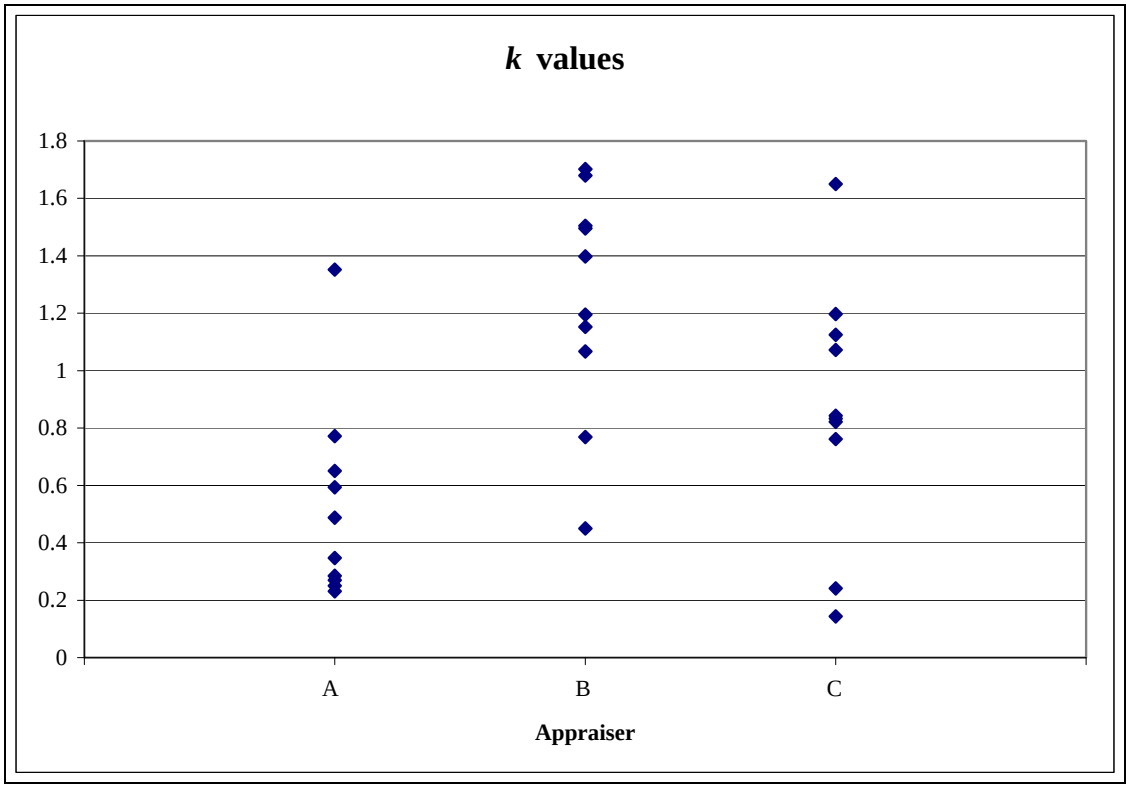


Figure IV-H 3: Dot diagram of *k* values.

APPENDICES

Appendix A

Analysis of Variance Concepts

The *GRR* numerical analysis can be done following the formulas in Table A 3. This is what is called the Analysis of Variance (ANOVA) table. The ANOVA table is composed of six columns:

- *Source* column is the cause of variation.
- *DF* column is the *degree of freedom* associated with the source.
- *SS* or *sum of squares* column is the deviation around the mean of the source.
- *MS* or *mean square* column is the sum of squares divided by degrees of freedom.
- *EMS* or *expected mean square* column determines the linear combination of variance components for each *MS*. The ANOVA table decomposes the total source of variation into four components: parts, appraisers, interaction of appraisers and parts, and replication error due to the gage/equipment repeatability.
- The *F-ratio* column is calculated only for the interaction in a MSA ANOVA; it is determined by the mean square of interaction divided by mean square error.

The estimate of variance components for each source is given in Table A 1.[82]

	Variance Estimate
Equipment (*EV*)	$\tau^2 = MS_e$ [83]
Interaction (*INT*)	$\gamma^2 = \frac{MS_{AP} - MS_e}{r}$
Appraiser (*AV*)	$\omega^2 = \frac{MS_A - MS_{AP}}{nr}$
Part (*PV*)	$\sigma^2 = \frac{MS_P - MS_{AP}}{kr}$

Table A 1: Estimate of Variance Components

Since each mean square is a sample quantity subject to sampling variation and the computations involve **differences of mean squares**, then negative variance component estimates are possible. This is a bit of a problem since the "master" variance components are equal or close to zero or have a small

[82] In this table, all components of variation are assumed to be random effects.

[83] In this application of ANOVA to measurement systems analysis, the ANOVA error term equates to the MSA equipment variation, MS_E.

sample size. For analysis purposes, the negative variance component is set to zero.

Standard deviation is easier to interpret than variance because it has the same unit of measurement as the original observation. In practice, the basic measure of spread is given by 6 times the standard deviation.[84] Table A 2 shows the 6 sigma spread for a measure of repeatability called equipment variation (*EV*) and measure of reproducibility called appraiser variation (*AV*). If the interaction of part and appraiser is significant, then there exists a non-additive model and therefore an estimate of its variance components is given. The *GRR* in Table A 2 is the total of measurement system variation.

$EV = 6\sqrt{MS_e}$	Equipment Variation = Repeatability
$AV = 6\sqrt{\frac{MS_A - MS_{AP}}{nr}}$	Appraiser Variation = Reproducibility
$I_{AP} = 6\sqrt{\frac{MS_{AP} - MS_e}{r}}$	Interaction of Appraiser by Part
$GRR = \sqrt{(EV)^2 + (AV)^2 + (I_{AP})^2}$	Gage R&R
$PV = 6\sqrt{\frac{MS_P - MS_{AP}}{kr}}$	Part Variation

Table A 2: 6 Sigma Spread

In the additive model, the interaction is not significant and the variance components for each source is determined as follow: First, the sum of square of gage error (SS_e from Table A 3) is added to the sum of square of appraiser by part interaction (SS_{AP} from Table A 3) and which is equal to the sum of squares pooled (SS_{pool}) with ($nkr - n - k + 1$)[85] degrees of freedom. Then the SS_{pool} will be divided by the ($nkr - n - k + 1$) to calculate MS_{pool}. The 6 sigma spread limit then will be:

$$EV = 6\sqrt{MS_{pool}}$$

$$AV = 6\sqrt{\frac{MS_A - MS_{pool}}{nr}}$$

$$GRR = \sqrt{(EV)^2 + (AV)^2}$$

$$PV = 6\sqrt{\frac{MS_P - MS_{pool}}{kr}}$$

[84] This is the 99.73% range. See note on page vi.

[85] Where n = number of parts, k = number of appraisers and r = number of trials.

In order to determine if the interaction is significant, compute the *F* statistic of appraiser by part interaction (see Table A 3). Compare this *F* statistic to an upper percentage point of an *F* distribution with numerator and denominator degrees of freedom taken from the ANOVA (Table A 3).

In order to decrease the risk of falsely concluding that there is no interaction effect, choose a high significance level. Once the *GRR* has been determined then the *%GRR* can be calculated in relation to process performance.

$$SS_P = \sum_{i=1}^{n}\left(\frac{x_{i..}^2}{kr}\right) - \frac{x_{...}^2}{nkr}$$

$$SS_A = \sum_{j=1}^{k}\left(\frac{x_{.j.}^2}{nr}\right) - \frac{x_{...}^2}{nkr}$$

$$TSS = \sum_{i=1}^{n}\sum_{j=1}^{k}\sum_{m=1}^{r}\left(x_{ijm}^2\right) - \frac{x_{...}^2}{nkr}$$

$$SS_{AP} = \sum_{i=1}^{n}\sum_{j=1}^{k}\left(\frac{x_{ij.}^2}{r}\right) - \sum_{i=1}^{n}\left(\frac{x_{i..}^2}{kr}\right) - \sum_{j=1}^{k}\left(\frac{x_{.j.}^2}{nr}\right) + \frac{x_{...}^2}{nkr}$$

$$SS_e = TSS - [SS_A + SS_P + SS_{AP}]$$

Source	DF	SS	MS	F	EMS
Appraiser	$k-1$	SS_A	$MS_A = \frac{SS_A}{(k-1)}$		$\tau^2 + r\gamma^2 + nr\omega^2$
Parts	$n-1$	SS_P	$MS_P = \frac{SS_P}{(n\ -\ 1)}$		$\tau^2 + r\gamma^2 + kr\sigma^2$
Appraiser-by-Part	$(n-1)\,(k-1)$	SS_{AP}	$MS_{AP} = \frac{SS_{AP}}{(n\ -\ 1)\ (k\ -\ 1)}$	$\frac{MS_{AP}}{MS_e}$	$\tau^2 + r\gamma^2$
Equipment	$nk\,(r-1)$	SS_e	$MS_e = \frac{SS_e}{nk\ (r\ -\ 1)}$		τ^2
Total	$nkr-1$	TSS			Appraiser ~ $N(0, \omega^2)$ Parts ~ $N(0, \sigma^2)$ Appraiser x Part ~ $N(0, \gamma^2)$ Equipment ~ $N(0, \tau^2)$

Table A 3: Analysis of Variance (ANOVA)

Tables A 4 and A 5 show the ANOVA calculations for our example data from Figure III-B 15.

Source	DF	SS	MS	F	EMS
Appraiser	2	3.1673	1.58363	79.41*	$\tau^2 + 3\gamma^2 + 30\omega^2$
Parts	9	88.3619	9.81799	492.29*	$\tau^2 + 3\gamma^2 + 9\sigma^2$
Appraiser by Part	18	0.3590	0.01994	0.434	$\tau^2 + 3\gamma^2$
Equipment	60	2.7589	0.04598		τ^2
Total	89	94.6471			

* Significant at α = 0.05 level

Table A 4: Tabulated ANOVA Results

Since the calculated F value for the interaction (0.434) is less than the critical value of $F_{\alpha,18,60}$, the interaction term is pooled with the equipment (error) term. That is, the estimate of variance is based on the model without interaction.

Estimate of Variance	Std. Dev. (σ)	6 (σ)	% Total Variation	% Contribution
τ^2 = 0.039973 (Equipment)	0.199933	*EV* = 1.199598	18.4	3.4
ω^2 = 0.051455 (Appraiser)	0.226838	*AV* = 1.361028	20.9	4.4
γ^2 = 0 (Interaction)		*INT* = 0	0	0
GRR = 0.09143 ($\tau^2 + \gamma^2 + \omega^2$)	0.302373	*GRR* = 1.814238	27.9	7.8
σ^2 = 1.086447 (Part)	1.042327	*PV* = 6.253962	96.0	92.2
Total Variation	1.085	*TV* = 6.51	100.0	

Table A 5: Tabulated ANOVA Results

$$ndc = 1.41(PV/GRR) = 1.41(6.25/1.81) = 4.87 \cong 4$$

$$Total\ Variation\ (TV) = \sqrt{GRR^2 + PV^2}$$

$$\%\ of\ Total\ Variation = 100\left(\frac{6\sigma_{(components)}}{6\sigma_{(total)}}\right) = 100\left(\frac{\sigma_{(components)}}{\sigma_{(total)}}\right)$$

$$\%\ Contribution\ (Total\ Variance) = 100\left(\frac{6\sigma^2_{(components)}}{6\sigma^2_{(total)}}\right) = 100\left(\frac{\sigma^2_{(components)}}{\sigma^2_{(total)}}\right)$$

Appendix B

Impact of *GRR* on the Capability Index *Cp*

Formulas:

$$\sigma^2_{Obs} = \sigma^2_{Act} + \sigma^2_{Meas} \quad (1)$$

where *Obs* = the **observed** process variation
Act = the **actual** process variation
Meas = the **measurement** system variation

$$Cp_x = \frac{|U - L|}{6\sigma_x} \quad (2)$$

where *U, L* are the upper and lower specification values
x = *Obs* or *Act* as defined in (1)

$$GRR\% = GRR_p * 100\% \quad (3)$$

based on process variation:

$$GRR_p = \frac{k\sigma_{Meas}}{6\sigma_{Obs}} \quad (4)$$

Note: $GRR_p \leq 1$ since $\sigma^2_{Obs} \geq \sigma^2_{Meas}$ by equation (1)

based on the tolerance range:

$$GRR_p = \frac{k\sigma_{Meas}}{|U - L|} \quad (5)$$

In (4) and (5), *k* is normally taken to be 6.

Analysis:

$$Cp_{Obs} = Cp_{Act} * \frac{\sigma_{Act}}{\sigma_{Obs}}$$

$$= Cp_{Act} * \frac{\sqrt{\sigma^2_{Obs} - \sigma^2_{Meas}}}{\sigma_{Obs}} \quad \text{using (1)}$$

with *GRR* based on the process variation

$$Cp_{Obs} = Cp_{Act} * \frac{\sigma_{Obs}\sqrt{1 - GRR^2}}{\sigma_{Obs}} \quad \text{using (4)}$$

$$= Cp_{Act} * \sqrt{1 - GRR^2} \quad (6)$$

or

$$Cp_{Act} = \frac{Cp_{Obs}}{\sqrt{1 - GRR^2}} \qquad (6')$$

with *GRR* based on the tolerance range

$$GRR = \frac{1}{Cp_{Obs}} * \frac{\sigma_{Meas}}{\sigma_{Obs}} \qquad \text{using (2) and (5)}$$

consequently

$$Cp_{Obs} = Cp_{Act} * \sqrt{1 - \left(Cp_{Obs} * GRR\right)^2} \qquad (7)$$

and

$$Cp_{Act} = \frac{Cp_{Obs}}{\sqrt{1 - \left(Cp_{Obs} * GRR\right)^2}} \qquad (7')$$

Graphical Analysis

Based on (6), the family of lines for Cp_{Obs} with respect to Cp_{Act} is:

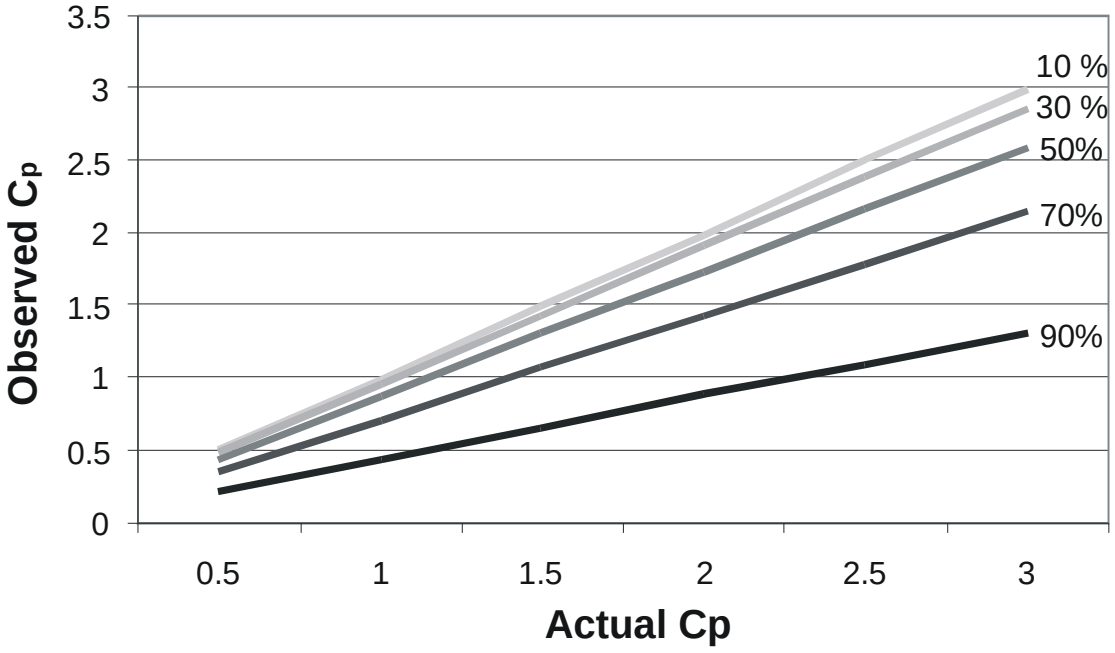


Figure B 1: Observed vs. Actual *Cp* (process based)

Actual	*GRR*							
	10%	20%	30%	40%	50%	60%	70%	90%
	Observed *Cp* with *Cp* based on Process Range							
1.3	1.29	1.27	1.24	1.19	1.13	1.04	0.93	0.57
	Observed *Cp* with *Cp* based on the Tolerance							
1.3	1.29	1.26	1.20	1.11	0.99	0.81	0.54	never

Table B 1: Comparison of Observed to Actual *Cp*

Based on (7'), the family of lines for Cp_{Act} with respect to Cp_{Obs} is:

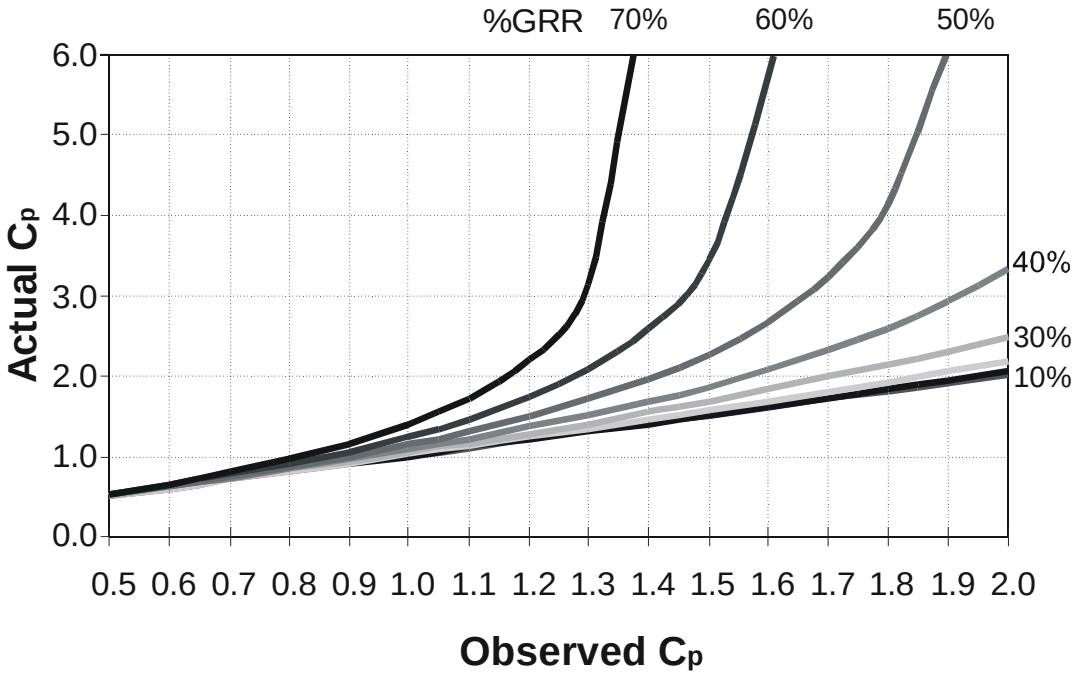


Figure B 2: Observed vs. Actual *Cp* (tolerance based)

Appendix C

Table C 1: d_2^* Table

Values associated with the Distribution of the Average Range

Number of Subgroups (*g*)	Subgroup Size (*m*) 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.0	2.0	2.9	3.8	4.7	5.5	6.3	7.0	7.7	8.3	9.0	9.6	10.2	10.8	11.3	11.9	12.4	12.9	13.4
	1.41421	1.91155	2.23887	2.48124	2.67253	2.82981	2.96288	3.07794	3.17905	3.26909	3.35016	3.42378	3.49116	3.55333	3.61071	3.66422	3.71424	3.76118	3.80537
2	1.9	3.8	5.7	7.5	9.2	10.8	12.3	13.8	15.1	16.5	17.8	19.0	20.2	21.3	22.4	23.5	24.5	25.5	26.5
	1.27931	1.80538	2.15069	2.40484	2.60438	2.76779	2.90562	3.02446	3.12869	3.22134	3.30463	3.38017	3.44922	3.51287	3.57156	3.62625	3.67734	3.72524	3.77032
3	2.8	5.7	8.4	11.1	13.6	16.0	18.3	20.5	22.6	24.6	26.5	28.4	30.1	31.9	33.5	35.1	36.7	38.2	39.7
	1.23105	1.76858	2.12049	2.37883	2.58127	2.74681	2.88628	3.00643	3.11173	3.20526	3.28931	3.36550	3.43512	3.49927	3.55842	3.61351	3.66495	3.71319	3.75857
4	3.7	7.5	11.2	14.7	18.1	21.3	24.4	27.3	30.1	32.7	35.3	37.7	40.1	42.4	44.6	46.7	48.8	50.8	52.8
	1.20621	1.74989	2.10522	2.36571	2.56964	2.73626	2.87656	2.99737	3.10321	3.19720	3.28163	3.35815	3.42805	3.49246	3.55183	3.60712	3.65875	3.70715	3.75268
5	4.6	9.3	13.9	18.4	22.6	26.6	30.4	34.0	37.5	40.8	44.0	47.1	50.1	52.9	55.7	58.4	61.0	63.5	65.9
	1.19105	1.73857	2.09601	2.35781	2.56263	2.72991	2.87071	2.99192	3.09808	3.19235	3.27701	3.35372	3.42381	3.48836	3.54787	3.60328	3.65502	3.70352	3.74914
6	5.5	11.1	16.7	22.0	27.0	31.8	36.4	40.8	45.0	49.0	52.8	56.5	60.1	63.5	66.8	70.0	73.1	76.1	79.1
	1.18083	1.73099	2.08985	2.35253	2.55795	2.72567	2.86680	2.98829	3.09467	3.18911	3.27392	3.35077	3.42097	3.48563	3.54522	3.60072	3.65253	3.70109	3.74678
7	6.4	12.9	19.4	25.6	31.5	37.1	42.5	47.6	52.4	57.1	61.6	65.9	70.0	74.0	77.9	81.6	85.3	88.8	92.2
	1.17348	1.72555	2.08543	2.34875	2.55460	2.72263	2.86401	2.98568	3.09222	3.18679	3.27172	3.34866	3.41894	3.48368	3.54333	3.59888	3.65075	3.69936	3.74509
8	7.2	14.8	22.1	29.2	36.0	42.4	48.5	54.3	59.9	65.2	70.3	75.2	80.0	84.6	89.0	93.3	97.4	101.4	105.3
	1.16794	1.72147	2.08212	2.34591	2.55208	2.72036	2.86192	2.98373	3.09039	3.18506	3.27006	3.34708	3.41742	3.48221	3.54192	3.59751	3.64941	3.69806	3.74382
9	8.1	16.6	24.9	32.9	40.4	47.7	54.5	61.1	67.3	73.3	79.1	84.6	90.0	95.1	100.1	104.9	109.5	114.1	118.5
	1.16361	1.71828	2.07953	2.34370	2.55013	2.71858	2.86028	2.98221	3.08896	3.18370	3.26878	3.34585	3.41624	3.48107	3.54081	3.59644	3.64838	3.69705	3.74284
10	9.0	18.4	27.6	36.5	44.9	52.9	60.6	67.8	74.8	81.5	87.9	94.0	99.9	105.6	111.2	116.5	121.7	126.7	131.6
	1.16014	1.71573	2.07746	2.34192	2.54856	2.71717	2.85898	2.98100	3.08781	3.18262	3.26775	3.34486	3.41529	3.48016	3.53993	3.59559	3.64755	3.69625	3.74205
11	9.9	20.2	30.4	40.1	49.4	58.2	66.6	74.6	82.2	89.6	96.6	103.4	109.9	116.2	122.3	128.1	133.8	139.4	144.7
	1.15729	1.71363	2.07577	2.34048	2.54728	2.71600	2.85791	2.98000	3.08688	3.18174	3.26690	3.34406	3.41452	3.47941	3.53921	3.59489	3.64687	3.69558	3.74141
12	10.7	22.0	33.1	43.7	53.8	63.5	72.6	81.3	89.7	97.7	105.4	112.7	119.9	126.7	133.3	139.8	146.0	152.0	157.9
	1.15490	1.71189	2.07436	2.33927	2.54621	2.71504	2.85702	2.97917	3.08610	3.18100	3.26620	3.34339	3.41387	3.47879	3.53861	3.59430	3.64630	3.69503	3.74087
13	11.6	23.8	35.8	47.3	58.3	68.7	78.6	88.1	97.1	105.8	114.1	122.1	129.8	137.3	144.4	151.4	158.1	164.7	171.0
	1.15289	1.71041	2.07316	2.33824	2.54530	2.71422	2.85627	2.97847	3.08544	3.18037	3.26561	3.34282	3.41333	3.47826	3.53810	3.59381	3.64582	3.69457	3.74041
14	12.5	25.7	38.6	51.0	62.8	74.0	84.7	94.9	104.6	113.9	122.9	131.5	139.8	147.8	155.5	163.0	170.3	177.3	184.2
	1.15115	1.70914	2.07213	2.33737	2.54452	2.71351	2.85562	2.97787	3.08487	3.17984	3.26510	3.34233	3.41286	3.47781	3.53766	3.59339	3.64541	3.69417	3.74002
15	13.4	27.5	41.3	54.6	67.2	79.3	90.7	101.6	112.1	122.1	131.7	140.9	149.8	158.3	166.6	174.6	182.4	190.0	197.3
	1.14965	1.70804	2.07125	2.33661	2.54385	2.71290	2.85506	2.97735	3.08438	3.17938	3.26465	3.34191	3.41245	3.47742	3.53728	3.59302	3.64505	3.69382	3.73969
16	14.3	29.3	44.1	58.2	71.7	84.5	96.7	108.4	119.5	130.2	140.4	150.2	159.7	168.9	177.7	186.3	194.6	202.6	210.4
	1.14833	1.70708	2.07047	2.33594	2.54326	2.71237	2.85457	2.97689	3.08395	3.17897	3.26427	3.34154	3.41210	3.47707	3.53695	3.59270	3.64474	3.69351	3.73939
17	15.1	31.1	46.8	61.8	76.2	89.8	102.8	115.1	127.0	138.3	149.2	159.6	169.7	179.4	188.8	197.9	206.7	215.2	223.6
	1.14717	1.70623	2.06978	2.33535	2.54274	2.71190	2.85413	2.97649	3.08358	3.17861	3.26393	3.34121	3.41178	3.47677	3.53666	3.59242	3.64447	3.69325	3.73913
18	16.0	32.9	49.5	65.5	80.6	95.1	108.8	121.9	134.4	146.4	157.9	169.0	179.7	190.0	199.9	209.5	218.8	227.9	236.7
	1.14613	1.70547	2.06917	2.33483	2.54228	2.71148	2.85375	2.97613	3.08324	3.17829	3.26362	3.34092	3.41150	3.47650	3.53640	3.59216	3.64422	3.69301	3.73890
19	16.9	34.7	52.3	69.1	85.1	100.3	114.8	128.7	141.9	154.5	166.7	178.4	189.6	200.5	211.0	221.1	231.0	240.5	249.8
	1.14520	1.70480	2.06862	2.33436	2.54187	2.71111	2.85341	2.97581	3.08294	3.17801	3.26335	3.34066	3.41125	3.47626	3.53617	3.59194	3.64400	3.69280	3.73869
20	17.8	36.5	55.0	72.7	89.6	105.6	120.9	135.4	149.3	162.7	175.5	187.8	199.6	211.0	222.1	232.8	243.1	253.2	263.0
	1.14437	1.70419	2.06813	2.33394	2.54149	2.71077	2.85310	2.97552	3.08267	3.17775	3.26311	3.34042	3.41103	3.47605	3.53596	3.59174	3.64380	3.69260	3.73850
d_2	1.12838	1.69257	2.05875	2.32593	2.53441	2.70436	2.8472	2.97003	3.07751	3.17287	3.25846	3.33598	3.40676	3.47193	3.53198	3.58788	3.64006	3.68896	3.735
cd	0.876	1.815	2.7378	3.623	4.4658	5.2673	6.0305	6.7582	7.4539	8.1207	8.7602	9.3751	9.9679	10.5396	11.0913	11.6259	12.144	12.6468	13.1362

Table entries: 1st line of each cell is the degrees of freedom (ν) and the 2nd line of each cell is d_2^*; d_2 is the infinity value of d_2^*; additional values of ν can be built up from the constant difference, *cd*.

Note: The notation used in this table follows that of Acheson Duncan, *Quality Control and Industrial Statistics*, 5th edition, McGraw-Hill, 1986.

$\nu\left(\overline{R}/d_2^*\right)^2/\sigma^2$ is distributed approximately as a χ^2 distribution with ν degrees of freedom where $\overline{R}$ is the average range of *g* subgroups of size *m*

Appendix D

Gage R Study

Application:

- Offers a preliminary determination of short-term gage repeatability only.
- May be used for screening before shipment from gage supplier.
- May be used in pre-production when part availability is minimal.
- May be used during gage development – e.g., for quickly comparing different clamping locations; comparing different methodologies.
- **This method CANNOT be used for final gage acceptance without other more complete and detailed MSA methods.**

Assumptions:

- Process stability and variation cannot be known at this point in time.
- Linearity and bias are not issues.
- Reproducibility is not considered here. The purpose is to focus on gage repeatability alone.
- *I/MR* (Individual/Moving Range) chart will provide minimal assessment of stability.

Analyze by:

Take one part, one operator; place part in fixture, measure; take part out of fixture; repeat this 9 more times with the same part and same operator.

Plot data on *I/MR* chart; assess stability. If data appears unstable, perform corrective action.[86]

If stable, calculate sigma of individuals by using either *s* for all readings or $\overline{MR}/d_2^*$; multiply by 6; divide this value by the characteristic tolerance; multiply by 100%. Review %Repeatability against previously established gage acceptability standards, or use for comparative purposes during gage development.

[86] Some judgment may be needed here, as 10 subgroups of individual data is insufficient to establish stability; however, obvious instabilities may still be assessed and provide value to the analysis.

Appendix E

Alternate *PV* Calculation Using Error Correction Term

PV has been defined here as $\sqrt{TV^2 - GRR^2}$. Since this definition of part variation can (does) include *EV*, there may be times when it is important to extract the *EV* influence on *PV*. This may be done by the following formula (note the similarity to the *AV* formula where *EV* influences are extracted).

$$PV = \sqrt{(R_P \times K_3)^2 - \left[\frac{EV^2}{k \times r}\right]}$$

where R_P = range of the part averages, k = # of appraisers, r = # trials.

Recognition of this method of calculating *PV* was published in 1997.[87] This method is presented here as a more statistically correct alternative to the commonly accepted definition of *PV* historically used in this manual. Generally, when *EV* contaminates *PV*, it does so to the extent of only a percentage point or two.

[87] "Reliable Data is an Important Commodity," Donald S. Ermer and Robin Yang E-Hok, University of Wisconsin, Madison, published in *The Standard*, ASQ Newsletter of the Measurement Quality Division, Vol 97-1, Winter, 1997.

Appendix E
Alternate *PV* Calculation Using Error Correction Term

Appendix F

P.I.S.M.O.E.A. Error Model

Similar to all processes, a measurement system is impacted by random and systematic sources of variation. These sources of variation are due to common and special (chaotic) causes. In order to understand, control and improve a measurement system, the potential sources of variation ought to first be identified. Although the specific causes will depend on the situation, a general error model can be used to categorize sources of variation for any measurement system. There are various methods of presenting and categorizing these sources of variation using simple cause & effect, matrix, or tree diagrams.

The acronym **P.I.S.M.O.E.A.**[88] represents another useful model for defining a measurement system by its basic sources of variation. It is not the only model, but does support universal application.

Error Source		Alias or Component	Factor or Parameter
P	**Part**	Production part, sample, measurand, Unit Under Test (UUT), artifact, check standard	Unknown
I	**Instrument**	Gage, unit of M&TE, master gage, measuring machine, test stand	Means of comparison
S	**Standard**	Scale, reference, artifact, check standard, intrinsic standard, consensus, Standard Reference Materials (SRM), class, acceptance criteria	Known value accepted as *truth* *, reference value, or acceptance criteria
M	**Method**	On-the-job training, verbal, work instruction, control plan, inspection plan, test program, part program	How
O	**Operator**	Appraiser, calibration or test technician, assessor, inspector	Who
E	**Environment**	Temperature, humidity, contamination, housekeeping, lighting, position, vibration, power, Electromagnetic Interference (EMI), noise, time, air	Conditions of measurement, noise
A	**Assumptions**	Statistical, operational, calibration, constants, handbook values, thermal stability, modulus of elasticity, laws of science	Criteria, constant, or supposition for reliable measurement

* Actual or physical *true values* are unknown

The alias error source varies with the measurement application and industry. These are common examples. However, the parameter and characteristic effect is always the same.

[88] P.I.S.M.O.E.A. was originally developed by Mr. Gordon Skattum, Senior ASQ CQE, metrologist and Director of Integrated Manufacturing for Rock Valley College Technology Center.

All methods of Measurement Systems Analysis are ***quality tools***. All quality tools are based on assumptions. If you violate the assumptions, the tool becomes unpredictable at best and could lead to a false conclusion at worst.

The typical statistical assumptions of a Gage R&R study, include: normal process, random and independent trials, stable, and test-retest criteria. When one or more assumption is violated (e.g., non-normal measurement process, operator bias) the tool and analysis ultimately become unstable, confusing, and misleading. %*GRR* evaluations for product and process control can be overestimated. There are also non-statistical assumptions related to measurement systems (e.g., calibration, operational, coefficients and rates of expansion, physical laws and constants). The measurement planner should be able to identify, control, correct, or modify the MSA method to accommodate significant violations for the assumptions in the measurement process.

Violation of the test-retest criteria is always a consideration for destructive test or in-process measurement systems where the feature is changing. The measurement planner ought to consider appropriate hybrids, modifications, or alternative techniques to the standard measurement system study.

Assumptions tend to contribute greater to total measurement variation when: 1) using more powerful methods or tools (e.g., ANOVA, regression analysis, designed experiments, probability predictions, and control charts), and 2) as measurement precision increases. High precision measurement applications must often plan for, and sometimes apply, correction for coefficients and thermal expansion, deformation, creep or other assumptions in the measurement process.

The greatest danger for a quality analyst or measurement planner regarding assumptions is that it is assumed they are insignificant or constant and therefore too often ignored.

Table F 1 demonstrates examples of the **PISMOEA** model for three different measurement situations.

Error Source	Typical Production, Automotive MSA	Automated In-Process or Test Stand	Calibration
P	Random production parts, entire process range	Production units, test samples, check standards, artifacts	Gage, UUT, test sample, measurand
I	Single type of production gage	DCC CMM, test stand	Master gage and equipment
S	Scale, master standard, or class; meets the "10 to 1 rule"	Scale and geometry, reference test standards	Master, reference, intrinsic or consensus, artifact
M	Standard Operating Procedures (S.O.P.), often verbal, may be documented; control plan	Documented S.O.P., DCC program or automated test cycle	Documented, formal calibration procedure
O	(2-3) typical, trained, who normally operate	Restricted test operator, specialized training and skill	Qualified technician, ISO 17025 proficiency evidence
E	Stable production and operating conditions	Often controlled	Control limits, optimized, a principle error source
A	Statistical, often ignored	Statistical, application specific	Cannot be assumed, a principle error source
Purpose	**Process control (SPC)**	**Product control, 100% inspection**	**Product control, calibration tolerance**

Table F 1: Examples of the PISMOEA **Model**

The degree of influence and contribution to measurement variation for specific error sources will depend on the situation. A matrix, cause and effect, or fault tree diagram will be a useful tool to identify and understand the dominant sources of measurement variation for control and improvement.

Measurement systems analysis starts by understanding the purpose and process of measurement. All sources of chaotic and illegitimate errors ought to be removed. A measurement study is a planned experiment that follows simple concepts: DEFINE significant error sources, FIX some, allow one or more CONTROL factors to change, MEASURE multiple trials, ANALYZE results, and TAKE ACTION.

Glossary

See the *Statistical Process Control (SPC) Reference Manual* for additional glossary definitions.

5.15 vs. 6 σ_{GRR} Multiplying Factor See note on page iv.

Accuracy The closeness of agreement between an observed value and the accepted reference value.

Analysis of Variance A statistical method (ANOVA) often used in designed experiments (*DOE*), to analyze variable data from multiple groups in order to compare means and analyze sources of variation.

Apparent Resolution The size of the least increment on the measurement instrument is the apparent resolution. This value is typically used in literature as advertisement to classify the measurement instrument. The number of data categories can be determined by dividing the size into the expected process distribution spread (6σ).
NOTE: The number of digits displayed or reported does not always indicate the resolution of the instrument. For example, parts measured as 29.075, 29.080, 29.095, etc., are recorded as five (5) digit measurements. However, the instrument may not have a resolution of .001 but rather .005.

Appraiser Variation The variation in average measurements of the same part (measurand) between different appraisers (operators) using the same measuring instrument and method in a stable environment. Appraiser variation (*AV*) is one of the common sources of measurement system variation (error) that results from differences in operator skill or technique using the same measurement system. Appraiser variation is commonly assumed to be the "reproducibility error" associated with a measurement system; this is not always true (see Reproducibility).

Bayes' Theorem A mathematical formula used for calculating conditional probabilities.

The probability of a hypothesis ***G*** conditional on a given body of data ***M*** is the ratio of the unconditional probability of the conjunction of the hypothesis with the data to the unconditional probability of the data alone
The probability of *G* conditional on *M* is defined as $P_M(G) = P(G \,\&\, M)/P(G)$, provided that both terms of this ratio exist and $P(G) > 0$

Bias The difference between the observed average of measurements (trials under repeatability conditions) and a reference value; historically referred to as *accuracy*. Bias is evaluated and expressed at a single point within the operating range of the measurement system.

Calibration A set of operations that establish, under specified conditions, the relationship between a measuring device and a traceable standard of known reference value and uncertainty. Calibration may also include steps to detect, correlate, report, or eliminate by adjustment any discrepancy in accuracy of the measuring device being compared.

Calibration Interval

A specified amount of time or set of conditions between calibrations during which the calibration parameters of a measuring device are considered valid.

Capability

An estimate of the combined variation of measurement errors (random and systematic) based on a short-term assessment of the measurement system.

Confidence Interval

An interval or range of values, calculated from sample data, that contains, with a (100 - α) degree of certainty, the population parameter of interest, e.g., the true population average.
α, called the Level of Significance, is the probability of committing a Type I error.
See Montgomery (1997) or Juran and Godfrey (1999) for calculation methods.

Control Chart

A graph of a process characteristic, based on sample measurements in time order, used to display the behavior of a process, identify patterns of process variation, assess stability, and indicate process direction.

- It displays the plotted values of some statistic gathered from that characteristic, a centerline, and one or two control limits.
- It minimizes the net economic loss from Type I and Type II errors.
- It has two basic uses: as a judgment to determine if a process has been operating in statistical control, and to aid in maintaining statistical control.

Data

A collection of observations under a set of conditions that may be either *variable* (a quantified value and unit of measure) or *discrete* (attribute or count data such as pass/fail, good/bad, go/no-go, etc.).

Designed Experiment

A planned study involving statistical analysis of a series tests in which purposeful changes are made to process factors, and the effects observed, in order to determine the relationship of process variables and improve the process.

Discrimination

Alias *smallest readable unit*, discrimination is the measurement resolution, scale limit, or smallest detectable unit of the measurement device and standard. It is an inherent property of gage design and reported as a unit of measurement or classification. The number of data categories is often referred to as the ***discrimination ratio*** since it describes how many classifications can be reliably distinguished given the observed process variation.

Distinct Data Categories

The number of data classifications or categories that can be reliably distinguished determined by the effective resolution of the measurement system and part variation from the observed process for a given application. See *ndc*.

Effective Resolution

The size of the data category when the total measurement system variation is considered is the effective resolution. This size is determined by the length of the confidence interval based on the measurement system variation. The number of distinct categories, *ndc*, can be determined by dividing the size into the expected process distribution spread. For the effective resolution, a standard estimate of this *ndc* (at the 97% confidence level) is 1.41[*PV/GRR*]. (See Wheeler, 1989, for an alternate interpretation.)

***F* ratio**	A statistic representing the mathematical ratio of the between-group mean square error to the within-group mean square error for a set of data used to assess the probability of random occurrence at a selected level of confidence.
Gage R&R (*GRR*)	An estimate of the combined variation of repeatability and reproducibility for a measurement system. The *GRR variance* is equal to the sum of within-system and between-system variances.
Histogram	A graphical representation (bar chart) of the frequency of grouped data to provide a visual evaluation of the data distribution.
In Control	State of a process when it exhibits only random, common cause variation (as opposed to chaotic, assignable, or special cause variation). A process operating with only random variation is statistically stable.
Independent	The occurrence of one event or variable has no effect on the probability that another event or variable will occur.
Independent and Identically Distributed	Commonly referred to as "*iid*". A homogeneous group of data which are independent and randomly distributed in one common distribution.
Interaction	A combined effect or outcome resulting from two or more variables that is significant. Non-additivity between appraiser and part. Appraiser differences depend on the part being measured.
Inter-rater agreement	**(**Also inter-rater reliability, or concordance) The degree of agreement among raters. It gives a score of how much homogeneity, or consensus, there is in the ratings given by the appraisers. There are a number of statistics which can be used to determine inter-rater reliability which are appropriate for different types of measurement. Some options are: joint-probability of agreement, Cohen's kappa and the related Fleiss' kappa, inter-rater correlation, concordance correlation coefficient and intra-class correlation**.**
Kappa (Cohen's)	A statistical measure of inter-rater agreement for qualitative (categorical) items. It takes into account the agreement occurring by chance.
Linearity	The difference in bias errors over the expected operating range of the measurement system. In other terms, linearity expresses the correlation of multiple and independent bias errors over the operating range.
Long-Term Capability	Statistical measure of the within-subgroup variation exhibited by a process over a long period of time. This differs from performance because it does not include the between-subgroup variation.
Measurand	The particular quantity or subject to be measured under specified conditions; a defined set of specifications for a measurement application.
Measurement system	A collection of instruments or gages, standards, operations, methods, fixtures, software, personnel, environment, and assumptions used to quantify a unit of measure or fix assessment to the feature characteristic being measured; the complete process used to obtain measurements.

Measurement System Error — The combined variation due to gage bias, repeatability, reproducibility, stability and linearity.

Metrology — The science of measurement.

ndc — Number of distinct categories. $1.41\left(PV/GRR\right)$

Non-replicable — The inability to make repeated measurements on the same sample or component due to the dynamic nature of the measurand.

Number of Distinct Categories — See *ndc*

Out-of-Control — State of a process when it exhibits chaotic, assignable , or special cause variation. A process that is out of control is statistically unstable.

Part Variation — Related to measurement systems analysis, part variation (*PV*) represents the expected part-to-part and time-to-time variation for a stable process.

Part-to-Part Variation — Piece-to-piece variation due to measuring different parts.

Performance — An estimate of the combined variation of measurement errors (random and systematic) based on a long-term assessment of the measurement system; includes all significant and determinable sources of variation over time.

Precision — The net effect of discrimination, sensitivity and repeatability over the operating range (size, range and time) of the measurement system. In some organizations precision is used interchangeability with repeatability. In fact, precision is most often used to describe the expected variation of repeated measurements over the range of measurement; that range may be size or time. The use of the more descriptive component terms is generally preferred over the term “precision”.

Probability — An estimate (in proportion or fraction), based on a particular distribution of collected data, describing the chance a specific event will occur. Probability estimates range between 0 (impossible event) to 1(sure thing). Set of conditions or causes working together to produce an outcome.

Process Control — Operational state when the purpose of measurement and decision criteria applies to real-time production to assess process stability and the measurand or feature to the natural process variation; the measurement result indicates the process is either stable and “in-control” or “out-of-control.”

Product Control — Operational state when the purpose of measurement and decision criteria is to assess the measurand or feature for conformance to a specification; the measurement result is either “in-tolerance” or “out-of-tolerance.”

Reference Value

A measurand value that is recognized and serves as an agreed upon reference or master value for comparison:

- A theoretical or established value based on scientific principles;
- An assigned value based on some national or international organization;
- A consensus value based on collaborative experimental work under the auspices of a scientific or engineering group; or
- For a specific application, an agreed upon value obtained using an accepted reference method.

A value consistent with the definition of a specific quantity and accepted, sometimes by convention, as appropriate for a given purpose.

NOTE: Other terms used synonymously with reference value
accepted reference value
accepted value
conventional value
conventional true value
assigned value
best estimate of the value
master value
master measurement

Regression Analysis

A statistical study of the relationship between two or more variables. A calculation to define the mathematical relationship between two or more variables.

Repeatability

The common cause, random variation resulting from successive trials under defined conditions of measurement. Often referred to as equipment variation (*EV*), although this is misleading. The best term for repeatability is *within*-system variation when the conditions of measurement are fixed and defined – fixed part, instrument, standard, method, operator, environment, and assumptions. In addition to *within*-equipment variation, repeatability will include all *within* variation from the conditions in the measurement error model.

Replicable

The ability to make repeated measurements on the same sample or component where there is no significant physical change to the measurand or measurement environment.

Replication

Multiple test trials under repeatable (identical) conditions.

Reproducibility

The variation in the average of measurements caused by a normal condition(s) of change in the measurement process. Typically, it has been defined as the variation in average measurements of the same part (measurand) between different appraisers (operators) using the same measuring instrument and method in a stable environment. This is often true for manual instruments influenced by the skill of the operator. It is not true, however, for measurement processes (i.e., automated systems) where the operator is not a major source of variation. For this reason, reproducibility is referred to as the average variation *between*-systems or *between*-conditions of measurement.

Resolution	May apply to *measurement* resolution or e*ffective* resolution The capability of the measurement system to detect and faithfully indicate even small changes of the measured characteristic. (See also discrimination.) The resolution of a measurement system is δ if there is an equal probability that the indicated value of any part which differs from a reference part by less than δ will be the same as the indicated value of the reference part. The resolution of a measurement system is impacted by the measurement instrument as well as other sources of variation of the total measurement system.
Scatter Diagram	A X-Y plot of data to assess the relationship between two variables.
Sensitivity	Smallest input signal that results in a detectable (discernable) output signal for a measurement device. An instrument should be at least as sensitive as its unit of discrimination. Sensitivity is determined by inherent gage design and quality, in-service maintenance, and operating condition. Sensitivity is reported in units of measurement.
Significance level	A statistical level selected to test the probability of random outcomes; also associated with the risk, expressed as the alpha (α) risk, that represents the probability of a decision error.
Stability	The absence of special causes of variation; the property of being in statistical control. Refers to both *statistical* stability of a measurement process and *measurement* stability over time. Both are vital for a measurement system to be adequate for its intended purpose. Statistical stability implies a predictable, underlying measurement process operating within common cause variation (in-control). Measurement stability (alias *drift*) addresses the necessary conformance to the measurement standard or reference over the operating life (time) of the measurement system.
Tolerance	Allowable deviation from a standard or nominal value that maintains fit, form, and function. See also Specification
Uncertainty	A parameter associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand (VIM); the range assigned to a measurement result that describes, within a defined level of confidence, the limits expected to contain the *true* measurement result. Uncertainty is a quantified expression of measurement reliability.
Unimodal	A contiguous group of data (distribution) that has one mode

Reference List

1. ASTM E691-99, *Standard Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method*, (www.astm.org).

2. ASTM E177-90a, *Standard Practice for Use of the Terms Precision and Bias in ASTM Test Method*, (www.astm.org).

3. ASTM E456-96, *Standard Terminology for Relating to Quality and Statistics*, (www.astm.org).

4. AT&T *Statistical Quality Control Handbook*, Delmar Printing Company, Charlotte, NC, 1984.

5. Baker, E. "Signal Detection Theory of Quality Control Inspector Performance", *Journal of Quality Technology*, Vol. 7, No.2, April 1975.

6. Bland, J. M., and Altman, D. G. (1986). Statistical methods for assessing agreement between two methods of clinical measurement. *Lancet* i, pp. 307–310.

7. Burdick, Richard K., and Larsen, Greg A., "Confidence Intervals on Measures of Variability in R&R Studies," *Journal of Quality Technology*, Vol. 29, No. 3, July 1997.

8. Cohen, J. (1960) "A coefficient for agreement for nominal scales" in *Education and Psychological Measurement*. Vol. 20, pp. 37–46.

9. Dataplot, National Institute of Standards and Technology, Statistical Engineering Division, (www.itl.nist.gov).

10. Deming, W. E., *Out of the Crisis*, Massachusetts Institute of Technology, 1982, 1986.

11. Deming, W. E., *The New Economics for Industry, Government, Education*, The MIT Press, 1994, 2000.

12. Duncan, A. J., *Quality Control and Industrial Statistics*, 5th ed., Richard D. Irwin, Inc., Homewood, Illinois, 1986.

13. Eagle, A. R., "A Method For Handling Errors in Testing and Measuring," *Industrial Quality Control*, March, 1954, pp. 10-14.

14. Ermer, Donald S., and E-Hok, Robin Yang, "Reliable Data is an Important Commodity," University of Wisconsin, Madison; published in The Standard, ASQ Newsletter of the Measurement Quality Division, Vol. 97-1, Winter 1997.

15. Everitt, B. (1996) *Making Sense of Statistics in Psychology* (Oxford University Press).

16. Fleiss, J. L. (1971) "Measuring nominal scale agreement among many raters" in *Psychological Bulletin*. Vol. 76, No. 5, pp. 378–382.

17. Foster, Lowell W., *GEO-METRICS: The Metric Application of Geometric Tolerancing*, Addison-Wesley Publishing Company, Reading, Massachusetts, 1993.

18. Gillespie, Richard, *A History of the Hawthorne Experiments*, Cambridge University Press, New York, 1991.

19. Grubbs, F. E., "On Estimating Precision of Measuring Instruments and Product Variability," *Journal of the American Statistical Association*, Vol. 43, 1948, pp.243-264.

20. Grubbs, F. E., "Errors of Measurement, Precision, Accuracy and the Statistical Comparison of Measuring Instruments," *Technometrics*, Vol. 15, February 1973, pp. 53-66.

21. Gruska, G. F., and Heaphy, M. S., "Measurement Systems Analysis," *TMI Conference Workshop*, Detroit, Michigan, 1987.

22. Guide to the Expression of Uncertainty in Measurement, ISO/IEC Guide 98:1995.

23. Hahn, J. H. and Nelson, W., "A Problem in the Statistical Comparison of Measuring Devices," *Technometrics*, Vol. 12., No. 1, February 1970, pp. 95-102.

24. Hamada, Michael, and Weerahandi, Sam, "Measurement System Assessment Via Generalized Inference," *Journal of Quality Technology*, Vol. 32, No. 3, July 2000.

25. Hayes, A. F. & Krippendorff, K. (2007). Answering the call for a standard reliability measure for coding data. *Communication Methods and Measures, 1*, 77-89.

26. Heaphy, M. S., et. al., "Measurement System Parameters," *Society of Manufacturing Engineering -IQ81-154*, Detroit, Michigan, 1981.

27. Hicks, Charles R.and Turner K.V. Jr. *Fundamental Concepts in the Design of Experiments*, Oxford University Press, New York, 1999.

28. *International Vocabulary of Basic and General Terms in Metrology* (abbr. VIM), ISO/IEC Guide 99:2007.

29. Ishikawa, Kaoru, *Guide to Quality Control*, Asian Productivity Organization, 1986.

30. Jaech, J. L., "Further Tests of Significance for Grubbs' Estimators," *Biometrics*, 27, December, 1971, pp. 1097-1101.

31. James, P. D., "Graphical Displays of Gage R&R Data," *AQC Transaction*, ASQC, 1991.

32. Krippendorff, K. (2004). *Content analysis: An introduction to its methodology*. Thousand Oaks, CA: Sage. pp. 219-250.

33. Lipson, C., and Sheth, N. J., *Statistical Design & Analysis of Engineering Experiment*, McGraw-Hill, New York, 1973.

34. Maloney, C. J., and Rostogi, S. C., "Significance Test for Grubbs' Estimators," *Biometrics*, 26, December, 1970. pp. 671-676.

35. Mandel, J., "Repeatability and Reproducibility," *Journal of Quality Technology*, Vol.4, No.2, April, 1972, pp.74-85.

36. McCaslin, J. A., and Gruska, G. F., "Analysis of Attribute Gage Systems," *ASQC Technical Conference Transactions*, 1976, pp. 392-399.

37. *Measurement Systems Analysis Reference Manual*, Third Edition, Chrysler, Ford, GM, Third Printing, 2002.

38. *Minitab User's Guide: Guide to Minitab: Release 15 - Minitab* , Minitab, Inc., 2007.

39. Montgomery, Douglas C., and Runger, George C., "Gauge Capability and Designed Experiments. Part I: Basic Methods; Part II: Experimental Design Models and Variance Component Estimation," *Quality Engineering*, Marcel Dekker, Inc., 1993.

40. Nelson, L., "Use of the Range to Estimate Variability," *Journal of Quality Technology*, Vol. 7, No.1, January, 1975, pp.46-48.

41. *Potential Failure Mode and Effects Analysis (FMEA) Reference Manual*, Fourth Edition, DaimlerChrysler, Ford, GM, 2008.

42. *Production Part Approval Process (PPAP)*, Fourth Edition, DaimlerChrysler, Ford, GM, 2008.

43. Saal, F.E., Downey, R.G. and Lahey, M.A (1980) "Rating the Ratings: Assessing the Psychometric Quality of Rating Data" in *Psychological Bulletin*. Vol. 88, No. 2, pp. 413–428.

44. Shrout, P. and Fleiss, J. L. (1979) "Intraclass correlation: uses in assessing rater reliability" in *Psychological Bulletin*. Vol. 86, No. 2, pp. 420–428.

45. Spiers, Beryl, "Analysis of Destructive Measuring Systems," *ASQC Quality Congress Transactions*, Toronto, 1989.

46. *Statistical Process Control Reference Manual (SPC)*, Second Edition, Chrysler, Ford, GM, 2005.

47. Thompson, W. A., Jr., "The Problem of Negative Estimates of Variance Components," *Annals of Mathematical Statistics*, No. 33, 1962, pp. 273-289.

48. Thompson, W. A., Jr., "Precision of Simultaneous Measurement Procedures", *American Statistical Association Journal*, June, 1963, pp. 474-479.

49. Traver, R. W., "Measuring Equipment Repeatability – The Rubber Ruler," *1962 ASQC Convention Transactions*, pp. 25-32.

50. Tsai, Pingfang, "Variable Gauge Repeatability and Reproducibility Study Using the Analysis of Variance Method," *Quality Engineering*, Marcel Dekker, Inc., 1988.

51. Uebersax, John S. (1987). "Diversity of decision making models and the measurement of interrater agreement" in *Psychological Bulletin*. Vol 101, pp. 140–146.

52. Vardeman, Stephen B., and VanValkenburg, Enid S., "Two-Way Random-Effects Analyses and Gauge R&R Studies," *Technometrics*, Vol. 41, No. 3, August 1999.

53. Wheeler, D. J., "Evaluating the Measurement Process When the Testing is Destructive," TAPPI, *1990 Polymers Lamination & Coatings Conference Proceedings*, (www.tappi.org search for document plc90805).

54. Wheeler, D. J., and Lyday, R. W., *Evaluating the Measurement Process*, SPC Press, Inc., Knoxville, Tennessee, 2006 .

Sample Forms

The reader has permission to reproduce the forms in this section for internal use only, not for commercial use or resale.

The forms in this section each represent a possible format for *GRR* data collection and reporting. They are not mutually exclusive to other types of formats which may contain the same information and accomplish the same results.

Gage Repeatability and Reproducibility Data Collection Sheet

Appraiser /Trial #	PART										AVERAGE
	1	2	3	4	5	6	7	8	9	10	
A 1											
2											
3											
Average											$\bar{X}_a$ =
Range											$\bar{R}_a$ =
B 1											
2											
3											
Average											$\bar{X}_b$ =
Range											$\bar{R}_b$ =
C 1											
2											
3											
Average											$\bar{X}_c$ =
Range											$\bar{R}_c$ =
Part Average											$\bar{\bar{X}}$ = R_p =
([$\bar{R}_a$ =] + [$\bar{R}_b$ =] + [$\bar{R}_c$ =]) / [# OF APPRAISERS =] =											$\bar{\bar{R}}$ =
$\bar{X}_{DIFF}$ = [Max $\bar{X}$ =] - [Min $\bar{X}$ =] =											$\bar{X}_{DIFF}$ =
* UCL_R = [$\bar{\bar{R}}$ =] × [D_4 =] =											

*D_4 = 3.27 for 2 trials and 2.58 for 3 trials. UCL_R represents the limit of individual *R*'s. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and re-average and recompute $\bar{\bar{R}}$ and the limiting value from the remaining observations.

Notes:_____

Gage Repeatability and Reproducibility Report

Part No. & Name: Gage Name: Date:
Characteristics: Gage No: Performed by:
Specifications: Gage Type:

From data sheet: $\bar{\bar{R}}$ = $\bar{X}_{DIFF}$ = R_p =

Measurement Unit Analysis | % **Total Variation (*TV*)**

Repeatability – Equipment Variation (*EV*)

$EV = \bar{\bar{R}} \times K_1$

= _____ × _____

= _____

Trials	K_1
2	0.8862
3	0.5908

%EV = 100 [*EV/TV*]

= 100 [_____/_____]

= _____**%**

Reproducibility – Appraiser Variation (*AV*)

$AV = \sqrt{(\bar{X}_{DIFF} \times K_2)^2 - (EV^2/(nr))}$

$= \sqrt{(\_\_\_\_\_ \times \_\_\_\_\_)^2 - (\_\_\_\_\_^2/(\_\_ \times \_\_))}$

= _____

n = parts *r = trials*

Appraisers	2	3
K_2	0.7071	0.5231

%AV = 100 [*AV/TV*]

= 100 [_____/_____]

= _____**%**

Repeatability & Reproducibility (*GRR*)

$GRR = \sqrt{EV^2 + AV^2}$

$= \sqrt{(\_\_\_\_\_^2 + \_\_\_\_\_^2)}$

= _____

%GRR= 100 [*GRR/TV*]

= 100 [_____/_____]

= _____**%**

Parts	K_3
2	0.7071
3	0.5231
4	0.4467
5	0.4030
6	0.3742
7	0.3534
8	0.3375
9	0.3249
10	0.3146

Part Variation (*PV*)

$PV = R_p \times K_3$

= _____ × _____

= _____

% PV = 100 [*PV/TV*]

= 100 [_____/_____]

= _____**%**

Total Variation (*TV*)

$TV = \sqrt{GRR^2 + PV^2}$

$= \sqrt{(\_\_\_\_\_^2 + \_\_\_\_\_^2)}$

= _____

ndc = $1.41\left(PV/GRR\right)$

= 1.41(_____/_____)

= _____

For information on the theory and constants used in the form see *MSA Reference Manual*, Fourth edition.

Index

M.S.A. Manual User Feedback Process

Consistent with the concept of continual improvement, this automotive industry measurement systems analysis (MSA) manual is being subjected to a formal periodic review/revision process. In line with the concept of customer satisfaction, this review will entail consideration of not only any applicable vehicle manufacturer requirement changes from year to year but also of feedback from users of the manual for the purpose of making it more value-added and effective to the automotive industry and user communities. Accordingly, please feel free to offer, in writing, your feedback comments, both pro and con, relative to the manual's understandability, "user-friendliness," etc., in the area indicated below. Please indicate specific manual page numbers where appropriate. Forward your feedback to the address indicated below:

Your Name _____

Representing _____
Company/Division Name

Address _____
_____
_____

Phone ___(____)_____

Please list your top three automotive customers and their locations.

_____ _____
Customer Location

_____ _____
Customer Location

_____ _____
Customer Location

Feedback Comments (attach additional sheets if needed)

_____
_____
_____
_____
_____

Send Comments To:
Automotive Industry Action Group
Suite 200
26200 Lahser Road
Southfield, Michigan 48033
MSA, 4th

Please access www.aiag.org to submit your feedback electronically.

MSA (Measurement Systems Analysis) 4th Edition Errata Sheet

Analysis of Results – Numerical

5) Compute the average bias of the n readings.

$$avg\ bias = \frac{\sum_{i=1}^{n} bias_i}{n}$$

6) Compute the repeatability standard deviation (see also Gage Study, Range Method, below):

$$\sigma_{repeatability} = \sigma_r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

If a *GRR* study is available (and valid), the repeatability standard deviation calculation should be based on the study results.

7) Determine if the repeatability is acceptable by calculating the

$$\%EV = 100\,[EV/TV] = 100\,[\sigma_{repeatability}/TV]$$

Where the total variation (*TV*) is based on the expected process variation (preferred) or the specification range divided by 6 (see also *GRR* study below).

If the %*EV* is large (see Chapter II, section D), then the measurement system variation may be unacceptable. Since the bias analysis assumes that the repeatability is acceptable, continuing the analysis with a measurement system with a large %*EV* will lead to misleading and confusing results.

8) Determine the *t* statistic for the bias: [34]

$$\sigma_b = \sigma_r / \sqrt{n}$$

$$t\ statistic = t_{bias} = \frac{average\ bias}{\sigma_b}$$

9) Bias is acceptable (statistically zero) at the α level if

- the p-value associated with t_{bias} is more than α; or

[34] The uncertainty for bias is given by σ_b.